MATH31B: Week 5

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Question 1. Write the following in summation notation

(a) $\dfrac{1}{2\cdot 3}+\dfrac{2}{3.4}+\cdots+\dfrac{n}{(n+1)(n+2)}$

(b) $3x^2+4x^3+\cdots 30x^{29}$

(c) $7x^6+9x^8+11x^{10}+\cdots 31x^{30}$

Question 2. Find the $2n$-th degree Taylor polynomial of $\cos(x)$ around the point $a=0$ and write it in summation notation.

Question 3. For the following functions f, find a value K such that for the given n and interval, $|f^{(n)}(x)|\leq K$ for all x in that interval.

(a) $f(x)=x^4$, $n=3$ on $[0,1]$

(b) $f(x)=\dfrac{1}{x}$, $n=4$ on $[1,2]$

(c) $f(x)=\cos(x)$ for all n and all $x\in\mathbb{R}$.

Question 4. Use the error bound for the Taylor polynomial to find error bounds for the following:

(a) $|f(0.1)-T_7(0.1)|$ where $f(x)=e^x$ and T_7 is centred at $a=0$.

(b) $|f(4.3)-T_3(4.3)|$ where $f(x)=x^{-1/2}$ and T_3 is centred at $a=4$.

Question 5. Use the error bound for the taylor polynomial to find a value for n such that $|\cos(0.1)-T_n(0.1)|\leq 10^{-7}$ holds. Here T_n is centred at $a=0$.

Question 6. Evaluate $\displaystyle\int\frac{18}{(x+1)(x^2+9)}dx$.