MATH 31B: Week 2

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Questions

Question 1. Compute without calculator:

(a) $\arcsin(\sin\frac{\pi}{3})$

(b) $\arcsin(\sin\frac{4\pi}{3})$

(c) $\arctan(\tan\frac{3\pi}{4})$

Solution to Question 1.

(a) Since the range of arcsin is $[\pi/2, \pi/2]$, we have that $\arcsin(\sin\frac{\pi}{3}) = \frac{\pi}{3}$.

(b) Since $\frac{4\pi}{3}$ is not in the range of arcsin, we can not use the previous solution. We want to find a value $y \in [\pi/2, \pi/2]$ such that $\sin(\frac{4\pi}{3}) = \sin(y)$, since this will then give us

$$\arcsin(\sin\frac{4\pi}{3}) = \arcsin(\sin y) = y.$$

In order to do this, we will use the identity $\sin(x) = \sin(\pi - x)$ (note, the general version of this identity is $\sin(x) = \sin((-1)^n(x - n\pi)),\ n \in \mathbb{Z}$ which can be used in the general case).

Now, we have that $\sin(\frac{4\pi}{3}) = \sin(\pi - \frac{4\pi}{3}) = \sin(-\frac{\pi}{3})$ and as $-\frac{\pi}{3}$ is in the range of arcsin we have that

$$\arcsin(\sin\frac{4\pi}{3}) = \arcsin(\sin\frac{-\pi}{3}) = \frac{-\pi}{3}.$$

(c) Similarly to the previous question. $\frac{3\pi}{4}$ is not in the range of arctan. However, we do have that $\tan(x)$ is π periodic ($\tan(x) = \tan(x + n\pi)$ for all $n \in \mathbb{Z}$) and so

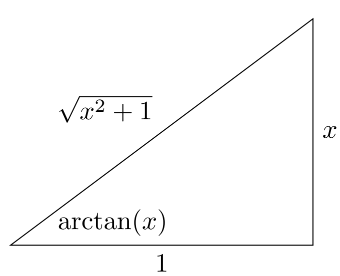
$$\arctan(\tan\frac{3\pi}{4}) = \arctan(\tan(\frac{3\pi}{4} - \pi)) = \arctan(\tan\frac{-\pi}{4}) = \frac{-\pi}{4}$$

Question 2. Compute without calculator:

1. $\cos(\arctan(x))$
2. $\cot(\sec^{-1}(x))$ for $x \geq 1$

Solution to Question 2.

1. We use a triangle method. We have the triangle



Hence, we have that $\cos(\arctan(x)) = \dfrac{1}{\sqrt{x^2+1}}$.

2. The triangle method can be used also. Another method is by trig identity $\tan^2(x) = \sec^2(x) - 1$.

$$\begin{aligned}\cot(\sec^{-1}(x)) &= \frac{1}{\tan(\sec^{-1}(x))} \\ &= \frac{1}{\pm\sqrt{\sec^2(\sec^{-1}(x)) - 1}} \\ &= \frac{1}{\pm\sqrt{x^2-1}}\end{aligned}$$

Note that since cot is positive between $[0, \pi/2]$ we take the potiive root. $\cot(\sec^{-1}(x)) = \dfrac{1}{\sqrt{x^2-1}}$

Question 3. Find the derivatives of the following functions:

(a) $\arcsin(e^x)$

(b) $\arccos(\ln(x))$

(c) $\sec^{-1}(t+1)$

(d) $\tan^{-1}\left(\dfrac{1+t}{1-t}\right)$.

Solution to Question 3.

(a) $\dfrac{e^x}{\sqrt{1-e^{2x}}}$

(b) $\dfrac{-1}{x\sqrt{1-\ln^2(x))}}$

(c) $\dfrac{1}{|t+1|\sqrt{(t+1)^2-1}}$

(d) $\dfrac{1}{1+t^2}$.

Question 4. Evaluate the following integrals:

(a) $\displaystyle\int \frac{dt}{\sqrt{1-16t^2}}$

(b) $\displaystyle\int \frac{dx}{x\sqrt{x^4-1}}$

(c) $\displaystyle\int \frac{\ln(cos^{-1}(x))dx}{(\cos^{-1}(x))\sqrt{1-x^2}}$.

Solution to Question 4.

(a) $\dfrac{1}{4}\arcsin(4t) + C$. Use substitution $u = 4t$.

(b) $\dfrac{1}{2}\sec^{-1}(x^2) + C$. Use substitution $u = x^2$.

(c) $-(\ln(\arccos(x)))^2 + C$.
Use substitution $u = \arccos(x)$.

Homework Questions

7.8.8, 7.8.57, 7.8.60, 7.9.55, 9.4.1, 9.4.15