Week 1: Limits

Problem 1 Solve the following problems which involve evaluating limits by plugging in points close to the limit.

(a) Let $f(x) = \dfrac{\sin x}{x}$. Evaluate $f(x)$ at the following values of x: 1, 0.1, 0.01. Use your answers to estimate $\lim_{x \to 0} f(x)$.

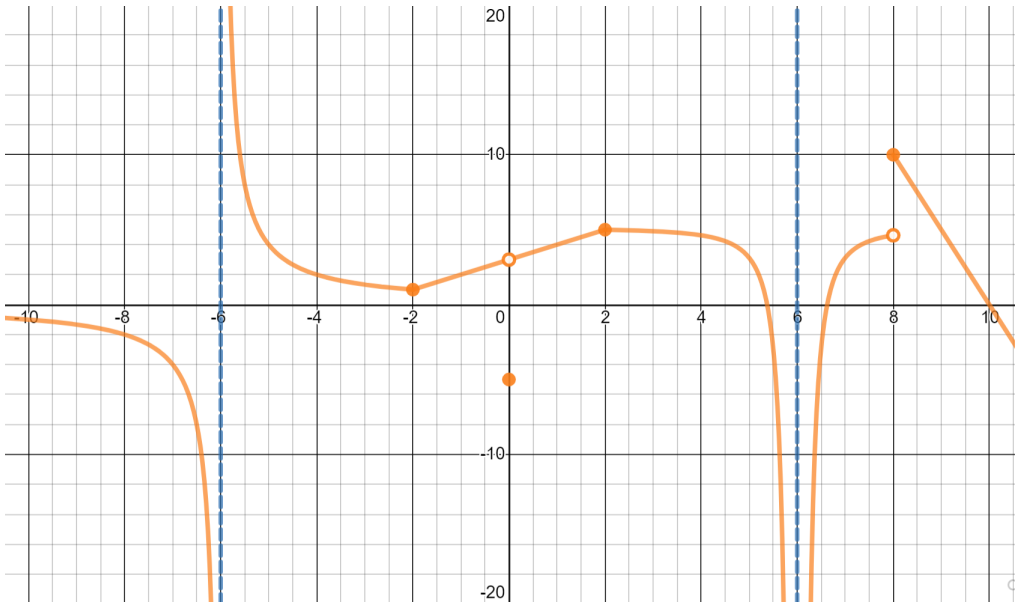
(b) Repeat part (a), but now with the following values of x: -1, -0.1, -0.01. Do you get the same estimate for $\lim_{x \to 0} f(x)$?

(c) Use the methods from parts (a) and (b) to estimate $\lim_{x \to 0} \dfrac{\sin(3x)}{x}$.

Problem 2 Answer the following questions involving discontinuities.

(a) Identify the following positions on the graph as an infinite discontinuity, a jump discontinuity, a removable discontinuity or not a discontinuity.

$x = -6$, $x = -2$, $x = 0$, $x = 2$, $x = 6$ and $x = 8$.



(b) Identify the discontinuities in the following piecewise function. For each discontinuity, say whether the graph is left-continuous, right-continuous or neither.

$$f(x) = \begin{cases} x + 20 & \text{for } x < -4 \\ 3x^3 & \text{for } -4 \leq x \leq -2 \\ \frac{1}{x^2-4} & \text{for } -2 < x < 2 \\ \sqrt{x-2} & \text{for } 2 \leq x < 6 \\ x^2 - 5x - 4 & \text{for } 6 \leq x \end{cases}$$

Problem 3 In the following problems, identify the limit if it exists. If not, evaluate the one-sided limits if they exist. If they do not exist, indicate if they can be expressed as $-\infty$ or $+\infty$.

(a) $\lim_{x\to 0} \dfrac{x^2+1}{x}$

(b) $\lim_{x\to 1} \dfrac{x^2+2x-5}{x^4-4x^3+5}$

(c) $\lim_{z\to -3} \dfrac{z+3}{z^2+4z+3}$

(d) $\lim_{\theta\to 0} \dfrac{\tan\theta - \sin^2\theta}{\sin^3\theta}$

(e) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$

(f) $\lim_{h\to 0} \dfrac{2(a+h)^2-2a^2}{h}$