Week 0: Functions, equations and graphs.

Problem 1 Get to know each other in your groups! Here are some things to talk about:

(a) Name and majors.

(b) What other classes are you taking this quarter?

(c) How do you plan to deal with remote learning and keeping motivated?

(d) Do you have any study strategies for maths?

(e) Do you know any good resources you can share with each other?

Problem 2 Build a function $y = f(x)$ valid on the real line such that the graph of f touches the points $P_1 = (1, 0)$, $P_2 = (2, 0)$ and $P_3 = (4, 0)$ in the xy-plane but *avoids* the following sets of points:

- $S_1 = \{(x, y) \in \mathbb{R}^2 \mid x = 1.5, -3 \leq y \leq 2\}$;
- $S_1 = \{(x, y) \in \mathbb{R}^2 \mid x = 3, y \neq 1\}$;.

Hint: Don't try and do this with one equation, but build your function piecewise.

Problem 3 Sketch the following equations:

(a) $y^2 = x^2$;

(b) $\dfrac{1}{y} = x^2 - 4$;

(c) $4x^2 - 9 = -y^2$.

Which ones determine functions and which don't? *Hint: If you are completelely lost, plug in points to see if they are on the graph or not and start building up a picture that way.*

Problem 4 For this problem, consider the function $f(x) = x^2 + x + 2$. We will construct the tangent line of this function at $x = 1$.

(a) Let $a > 0$. Find the equation of the secant line of the function at $x = 1$ and $x = 1 + a$. That is, find the equation of the line that connects the two points $(1, f(1))$ and $(1 + a, f(1 + a))$. Think of a as a 'little bit of x'.

(b) This equation for the line depends on a, so in particular the slope of this secant line depends on a. Write the slope of the secant line as a function of $a > 0$.

(c) What is the slope of the secant line when $a = 1, 0.1, 0.01$. Does this get closer and closer to a specific number?

(d) We have only defined the slope when $a > 0$. However, your function should still make sense when you insert $a = 0$. What do you get when you do this? Geometrically, what do you think this signifies? *Hint: If your function doesn't make sense, you might need to cancel something off*

(e) Try and do part Problem (4.a) when $a = 0$. Why can't we do this?

(f) What is the tangent line of the fucntion at $x = 1$?

At this point you hopefully have an idea now that something odd is happening in that we can get a reasonable answer by first approximating our problem by introducing a small change in one of our values (adding a to 1 — a 'little change in x') and then considering what happens when this approximation gets closer and closer to our original problem (a getting smaller and closer to zero) and mysteriously, this allows us to deal with problems that just plugging numbers into things doesn't give us. From a certain point of view, formalizing this behaviour is the point of differential calculus.