MATH 31A: Week 1

TA: Ben Szczesny Last updated: 2018/10/02

Discussion Questions

Question 1. Use the formal definition of limit to prove that $\lim_{x\to 3} 2x + 4 = 10$.

Question 2. In some cases, numerical investigations of limits can be misleading. Consider the function $f(x) = \cos\left(\frac{\pi}{x}\right)$.

(a) Evaluate $f(x)$ at $x = \pm\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots,$.

(b) Does $\lim_{x\to 0} f(x)$ exist?

Question 3. Evaluate the following limits assuming that $\lim_{x\to 2} f(x) = 2$ and $\lim_{x\to 2} g(x) = 5$.

(a) $\lim_{x\to 2} f(x)g(x)$

(b) $\lim_{x\to 2} \frac{g(x)}{f(x)}$

(c) $\lim_{x\to 2} 2g(x) - 3$

(d) $\lim_{x\to 2} \frac{4f(x)}{g(x) - 2}$

Question 4. Give an example where $\lim_{x\to 0}(f(x) + g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

Question 5. Determine the points of discontinuity (removable, jump, infinite or none of these) and whether the function is left- or right-continuous.

(a) $f(x) = \left\lfloor \frac{x}{2} \right\rfloor$

(b) $f(x) = \dfrac{x+1}{4x-2}$

(c) $f(x) = \begin{cases} x^2 & \text{for } x \leq 1 \\ 2 - x & \text{for } x > 1 \end{cases}$

Question 6. Suppose that $\lim_{t\to 3} tg(t) = 12$. Show that $\lim_{t\to 3} g(t)$ exists and is equal to 4.