

Math210B: Week 5

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Question 1. Show that $R/I \otimes R/J = R/(I+J)$ where R is a commutative ring and I, J are ideals. Given this, why is it true that $\mathbb{Z}/(a) \otimes \mathbb{Z}/(b) = \mathbb{Z}/(\gcd(a, b))$?

Question 2. Let S be a multiplicative set of R and let M be an R -module. Show that

1. For R -module A , show that

$$(S^{-1}A) \otimes_R M \cong S^{-1}(A \otimes_R M)$$

2. For $S^{-1}R$ -module N , then

$$(S^{-1}M) \otimes_{S^{-1}R} N \cong M \otimes_R N.$$

Question 3. Show that flatness is a local property.

Question 4. Show the following:

1. $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q} \cong \mathbb{Q} \otimes_{\mathbb{Q}} \mathbb{Q}$
2. $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \not\cong \mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$