Math 210B: Week 3

TA: Ben Szczesny Last updated: 2020/01/19

Question 1. The following are standard properties of localisations which you should prove if you haven't already.

1. let S be a multiplicative set of commutative ring R. Show there is a bijection between prime ideals of R that don't intersect S and prime ideals of $R[S^{-1}]$
2. Let $S \subseteq W$ be multiplicative sets in R. Show that $(R[S^{-1}])[\overline{W}^{-1}] \cong R[W^{-1}]$.

Question 2. Given $f \in R$, let $D(f) = Spec(R) \setminus V(f)$. This is called a basic open set in the spectrum of R. Let S be the set of all functions $g \in R$ that don't vanish on $D(f)$. Show S is multiplicatively closed and that $R_f = R[S^{-1}]$.

Question 3. The sets $D(f)$ are called basic open sets or disinguished open sets. Show that these form a basis for $Spec(R)$. Moreover, show that $Spec(R)$ is compact.

Question 4. let $f_1, \cdots, f_n$ be such such that $D(f_i)$ cover the $Spec(R)$. Prove the following: (note $D(f) \cap D(g) = D(fg)$.

1. (identity) if $g, h \in R$ such that $g = h$ in R_{f_i} for all i. Then $g = h$ in R.
2. (glueing) if g_i in R_{f_i} for all i such that $g_i = g_j$ in $R_{f_i f_j}$ for $i \neq j$. Then there exists a $g \in R$ such that $g = g_i$ in R_{f_i}.

The previous exercise shows that the localisations form something called a Sheaf on the distinguished basis and so a sheaf on $Spec(R)$. The localisations R_f should be viewed as the ring of functions over $D(f)$.