MTH210A: Week 8

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Area Exam Bonanza

Let's end the group theory section with a bunch of qual problems

Question 1. (Fall '11 Q2)

Let G be a nontrivial group and p a prime. If every subgroup $H \neq G$ has index divisible by p, prove the center of G is divisible by p. Note, I think the question wants to say for all nontrivial $H \neq G$.

Question 2. (Spring '07 G2)

How many subgroups of $\mathbb{Z}^n$ have index 5?

Question 3. (Fall '17 Q1)

Let G be a finite group, p a prime number and S a sylow p-subgroup of G. Let $N = \{g \in G \mid gSg^{-1} = S\}$. Let X and Y be subsets of $Z(S)$ such that there exists a $g \in G$ such that $gXg^{-1} = Y$. Show there exists an $n \in N$ such that $nxn^{-1} = gxg^{-1}$ for all $x \in X$.

Question 4. (Spring '13, Q1)

Let G be a free abelian group of rank r. Show that G has only finitely many subgroups of a given finite index n.

HINTS BELOW. FOLD HERE.

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1. We have seen many times that a p-group has nontrivial center. How is this proven and can we extend it to this question?
2. Every subgroup can be given by a surjective hom $\mathbb{Z}^n \to \mathbb{Z}/5\mathbb{Z}$. However, different homs can give the same subgroup, but homs up to some symmetry condition give the same subgroup. We talked last week how to count things of this type.
3. Consider Sylow p-subgroups in $Fix(X)$.
4. Same idea as Q2.