MATH210A: Week 3

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The first two questions directly use the Sylow Theorems, while the rest use Sylow and group actions similar to those used to prove the Sylow theorems in some way.

Question 1. Prove that there are no simple groups of order 18.

Question 2. Let P be a Sylow p-subgroup of finite group G. Suppose H is a p-subgroup of G and $H \subseteq N_G(P)$. Show that $H \subseteq P$.

Question 3. Let P be a Sylow p-subgroup of a finite group G. Denote by X the set of all Sylow p-subgroups of G and consider the group action of P on X via conjugation. Show that P is the unique fixed point of this action.

Question 4. Let n_p denote the number of Sylow p-subgroups of a finite group G. Prove that if G is simple, then $|G|$ divides $n_p!$ for each prime p in the factorisation of $|G|$. Hint: Consider the action of G on the set of Sylow p-subgroups via conjugation.

Question 5. (Qual S07.G1) Let G be a simple group that has an element of order 21. Show that every proper subgroup has index of at least 10. Note, this problem doesn't require any of the Sylow theorems. Instead think about left action on cosets.

Question 6. (Qual S14.5) Let G be a finite group that acts transitively on the set X. Given an $x \in X$, let P be a Sylow p-subgroup of the stabliser G_x. Show that $N_G(P)$ acts transitively on X^P.