MATH210A: Week 2

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In the following questions we will prove the Jordan-Hölder Theorem: For a finite nontrivial group G, there exists a composition series for G and any two composition series for G are equivalent.

Question 1. Prove any finite group G has a composition series. Moreover, use the bijection between subgroups of a quotient group G/H and subgroups of G containing the subgroup H to show that any subnormal series can be refined to a composition series.

Question 2. Let G be a finite group and $A, B \trianglelefteq G$ with $A \neq B$ and G/A, G/B simple groups. Show that

$$G/A \cong B/(A \cap B) \quad G/B \cong A/(A \cap B)$$

Question 3. Prove Jordan-Hölder. *Hint: Argue via induction on the length of the shortest composition series*

Now we will go through some applications of Jordan-Hölder

Question 4. Prove the fundamental theorem of arithmetic using Jordan-Hölder.

Question 5. What can you say about the composition factors of a finite abelian group G? What about the composition factors of a finite solvable group G?