

MATH210A: Week 10

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Question 1. Consider the free group functor $F : \mathbf{Set} \rightarrow \mathbf{Grp}$. Is this a left adjoint? a right adjoint?

Question 2. (Fall '18 Q7) Consider a functor $F : \mathcal{C} \rightarrow \mathcal{D}$ with a right adjoint G . Show that F is fully faithful if and only if the unit of adjunction $\eta : \text{id}_{\mathcal{C}} \rightarrow GF$ is an isomorphism.

Question 3. (Fall '17 Q10) Let \mathcal{C} be a category with finite products and $F : \mathcal{C}^2 \rightarrow \mathcal{C}$ the functor that takes pairs to their product.

1. Find a left adjoint to F .
 2. For \mathcal{C} the category of abelian groups, determine if F has a right adjoint.
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HINTS BELOW. FOLD HERE.

1. The forgetful functor $U : \mathbf{Grp} \rightarrow \mathbf{Set}$ is adjoint to it. For the other one, consider either limits or colimits.
2. By Yoneda, η being an isomorphism is the same thing as for all $X, X' \in \mathcal{C}$ and $\eta_X \circ : \text{hom}(X', X) \rightarrow \text{hom}(X', GF(X))$ being a bijection.
3. The category of abelian groups has finite biproducts. That is, finite products are also coproducts and vice versa.