Math 210C Homework 9

due 6/6/2013

Problem 1 *Using the character tables for cyclic groups, calculate the character tables for each of the three abelian groups of order* 8.

Problem 2 *Consider the dihedral group* $D_n = \langle x, y \mid x^2 = 1,\ y^n = 1,\ xyx = \ y^{-1}\rangle$ *with* $2n$ *elements.*

(a) Let $\epsilon = e^{2\pi i/n}$*, and for* $1 \le j < n/2$*, let* $A_j = \begin{pmatrix} \epsilon^j & 0 \\ 0 & \epsilon^{-j} \end{pmatrix}$ *and* $B_j = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. *Show that* $\rho_j : D_n \to GL(2\mathbb{C})$, $y^r x^s \mapsto (A_j)^r (B_j)^s)$ *gives an irreducible representation of* G. *Show that* ρ_i *and* ρ_j *are not equivalent for* $i \neq j$.

(b) For n *odd, complete the character table by lifting the irreducible characters of* $D_n/\langle y\rangle$ *to* D_n.

(c) For n *even, complete the character table by lifting the irreducible characters of* $D_n/\langle y^2\rangle$ *to* D_n.

Problem 3 *Find the character table of the group of the quaternions* $\{\pm 1,\ \pm i,\ \pm j,\ \pm k\}$.

Problem 4 *Let* V *be the standard representation of* S_3. *Find the decomposition of the representation* $V^{\otimes n}$ *using character theory.*

Problem 5 *If* V_1 *and* V_2 *are representations of the groups* G_1 *and* G_2, *the tensor product* $V_1 \otimes V_2$ *is a representation of* $G_1 \times G_2$ *by* $(g_1 \times g_2) \cdot (v_1 \otimes v_2) = g_1 v_1 \otimes g_2 v_2$. *To distinguish this from the case* $G_1 = G_2$, *we denote this by* $V_1 \boxtimes V_2$. *Let* χ_i *be the character of* V_i. *(a) Show that the character* χ *of* $V_1 \boxtimes V_2$ *is given by:* $\chi(g_1 \times g_2) = \chi_1(g_1)\chi_2(g_2)$. *(b) If* V_1 *and* V_2 *are irreducible, show that* $V_1 \boxtimes V_2$ *is irreducible and every irreducible representation of* $G_1 \times G_2$ *arises this way.*

Problem 6 *Show that the dimension of an irreducible representation of* G *divides the order of* G.

Problem 7 *Let* $G \to GL(V)$ *be a finite dimensional irreducible representation of a finite group* G *in a complex vector space* V. *Let* $\beta_1, \beta_2 : V \times V \to \mathbb{C}$ *be a pair of nonzero hermition (not necessarily positive definite)* G*-invariant forms on* V. *Prove that there exists a nonzero constant* $c \in \mathbb{R}$ *such that one has* $\beta_2(v_1, v_2) = c \cdot \beta_1(v_1, v_2)$ *for any* $v_1, v_2 \in V$.