Math 210C Homework 7

due 5/23/2013

Problem 1 *Give an example of a faithful representation of* D_8 *of degree* 3.

Problem 2 *Let* g *be an element of a finite group* G *such that* $\rho(g) = Id_V$ *for any irreducible complex representiaton* $\rho : G \to GL(V)$. *Show that* $g = e$ *is the identity of the group* G.

Problem 3 *Let* G *be an infinite group and* F *a field. Show that the group ring* FG *is not semi-simple.*

Problem 4 *Show that if* ρ_i, $i = 1, 2$, *is a representation acting on* V_i, *then* $\rho_1 \otimes \rho_2$ *and* $\rho_2 \otimes \rho_1$ *are isomorphic.*

Problem 5 *Let* ρ *be a representation of a group* G *that is not necessarily finite. Suppose* G *contains a normal subgroup* H *of finite index* $[G : H]$ *not divisible by* $charF$. *Show that if* $\rho|_H$ *is semi-simple, then* ρ *is semi-simple.*

Problem 6 *Let* p *be a prime and* C_p *the cyclic group of order* p.

(a) Describe the Artin-Wedderburn decompositions for kC_p *when* $k = \mathbb{Q}$.

(b) Describe the Artin-Wedderburn decompositions for kC_p *when* $k = \mathbb{R}$.

(c) Describe the Artin-Wedderburn decompositions for kC_p *when* $k = \mathbb{C}$.

Problem 7 *Describe the Artin-Wedderburn decomposition of* kG, *for* $G = C_2 \times C_2$ *and* k *of characteristic not* 2.

Problem 8 *Let* S_3 *be the group of permutations of* 3 *letters. Show that* kS_3 *is isomorphic to* $k \times k \times M_2(k)$, *whenever* $char(k) \neq 2, 3$.

Problem 9 *Let* V *and* W *be complex representations of* G. *Show that:*

(a) $\chi_{V \oplus W} = \chi_V + \chi_W$.

(b) $\chi_{V \otimes W} = \chi_V \cdot \chi_W$.

(c) $\chi_{V^*} = \bar{\chi}_V$.