Math 210B Homework 3

Question 1. Suppose $G =< x, y, z, w >$, considered as an additive group. Let $a = x - z + 2w$, $b = x - y + w$, $c = 3x - y - 2z + 5w$, and $d = 2x - 2y + 4w$. If $H =< a, b, c, d >$, determine the structure of G/H.

Question 2. Find all similarity classes of 4×4 matrices A over $\mathbb{Q}$ such that $A^2 \neq \pm A$ and $A^2 \neq I$ but $A^3 = A$.

Question 3. For a matrix $A \in M_n(F)$, F a field, let $C_A = \{B \in M_n(F) | \; AB = BA\}$.

(a) Find the minimum dimension m of C_A among all possible A.

(b) Determine the invariant factors of all A such that $\dim C_A = m$.

Question 4. Let p be a prime, let $S = \{p^n | \; n \geq 0\}$, and let $T = \mathbb{Z} \setminus p\mathbb{Z}$.

(a) Classify all finitely generated modules over $S^{-1}\mathbb{Z}$.

(b) Classify all finitely generated modules over $T^{-1}\mathbb{Z}$.

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Question 5. Classify all finitely generated modules over $\mathbb{Z}/n\mathbb{Z}$.

Question 6. Find the invariant factors of the factor group $\mathbb{Z}^3/N$, where N is generated by $(-4, 4, 2)$, $(16, -4, -8)$, and $(8, 4, 2)$.

Question 7. Let $\mathbb{F}_2$ be the field of 2 elements and let $R = \mathbb{F}_2[X]$. List, up to isomorphism, all R-modules with exactly 8 elements.

Question 8. Find the rational canonical form (or Frobenius normal form, i.e. block-diagonal with companion matrices on the diagonal), of the linear operator in $\mathbb{R}^3$ given by the matrix

$$\begin{pmatrix} -2 & 0 & 0 \\ -1 & -4 & -1 \\ 2 & 4 & 0 \end{pmatrix}.$$