

Math 210A Homework 10

Question 1. Show that the ring $R = \mathbb{Z}[x_1, x_2, \dots] = \mathbb{Z}[x_i]_{i \in \mathbb{N}}$ is a UFD. Is it noetherian?

Question 2.

(a) Suppose R is noetherian and $I \subset R$ is an ideal of R . Prove that R/I is noetherian.

(b) Suppose R is artinian and $I \subset R$ is an ideal of R . Prove that R/I is artinian.

(c) If $R[x]$ is noetherian, does R necessarily have to be noetherian as well?

Question 3.

(a) Let R be a noetherian ring and S a multiplicative subset. Prove $S^{-1}R$ is noetherian.

(b) Let R be an artinian ring and S a multiplicative subset. Prove $S^{-1}R$ is artinian.

Question 4. Prove that any artinian integral domain is a field.

Question 5. Give two examples of noetherian rings that are not artinian.

Question 6. Let R be the ring of 2×2 matrices $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ such that $a \in \mathbb{Z}$ and $b, c \in \mathbb{Q}$. Prove that R is right noetherian but not left noetherian.

Question 7. Let R be the ring of 2×2 matrices $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ such that $a \in \mathbb{Q}$ and $b, c \in \mathbb{R}$. Prove that R is right artinian but not left artinian.

This homework is not due and will not be graded.