1. For each of the following series, say (1) whether the series converges or diverges, (2) the specific criterion you used, and (3) if you used a comparison of some kind, the easier series with which you compared it. You may omit other details, such as whether you used the whole series or a tail. Example: "converges; comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$"

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{3}{n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4 3^n}{4^n}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{(\log n)^3}{n^{1.2}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\log n)^2}$

2. For each of the following series, say whether the series converges absolutely, converges conditionally, or diverges, and give a brief reason. Example: "Conditionally; converges by alternating series theorem, but abs vals diverge by comparison with harmonic."

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^4}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{\frac{1}{2}}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+1}{2n^2}$

(e) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n} \frac{n}{1.2^n}$

3. For each of the following functions, (1) give its Taylor/Maclaurin series about $x = 0$ **and** (2) find the radius of convergence. (You don't need to determine what happens at endpoints of the interval of convergence. In finding the series, you should start from either the definition of a Taylor/Maclaurin series or else your knowledge of geometric series. Show your reasoning.)

(a) $f(x) = \dfrac{1}{1-2x}$ (for $x \neq \frac{1}{2}$)

(b) $f(x) = e^{-2x}$

(c)[4 points] $f(x) = \log(x+1)$ (for $x > -1$).

4. Approximate $(1.1)^{4/3}$ by using a Taylor series about $x = 1$ through the $n = 2$ term and use the Taylor remainder to check that your answer is accurate to within .001. (You may leave your approximation as an unsimplified arithmetic expression, for example $1 + \frac{5}{7} + \frac{3\cdot 8}{5\cdot 343}$. Show your reasoning.)

5. (a)[2 points] Say how the Euler γ (gamma) constant is defined. (You don't need to give the approximate value.)

(b) [8 points] Prove **one** of the following. (Circle (i), (ii), or (iii).)

(i) If $\sum_{n=1}^{\infty} a_n$ converges then $a_n \to 0$ as $n \to \infty$.

(ii) Suppose $a_n > 0$ and $b_n > 0$ for all n. If $\frac{a_n}{b_n} \to C$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. (Do the case $C > 0$ only).

(iii) If $a_n > 0$ for all n and $\frac{a_{n+1}}{a_n} \to r < 1$, then $\sum_{n=1}^{\infty} a_n$ converges.