Homework 19
The Chain Rule

1. (800 #2) Find dz/dt when $z = x^2y^3$, $x = 1+\sqrt{t}$ and $y = 1-\sqrt{t}$

ans $-\dfrac{(1-t)(1-\sqrt{t})(1+5\sqrt{t})}{2\sqrt{t}}$

2. (800,#6) Find dw/dt for $w = \dfrac{x}{y} + \dfrac{y}{z}$, $x = \sqrt{t}$, $y = \cos(2t)$, $z = e^{-2t}$

ans $\dfrac{\sec(2t)}{2\sqrt{t}} + 2\sin(2t)(\sqrt{t}\sec^2(2t) - e^{2t}) + 2\cos(2t)e^{2t}$

3. (800, #8) Find $\dfrac{\partial z}{\partial s}$ *and* $\dfrac{\partial z}{\partial t}$ for $z = \sin(x)\cos(y)$, $x = (s-t)^2$, $y = s^2 - t^2$

$2(s-t)\cos((s-t)^2)\cos((s^2-t^2)) - 2s\sin((s-t)^2)\sin((s^2-t^2))$

$-2(s-t)\cos((s-t)^2)\cos((s^2-t^2)) + 2t\sin((s-t)^2)\sin((s^2-t^2))$

4. (800, #10) Find $\dfrac{\partial z}{\partial s}$ *and* $\dfrac{\partial z}{\partial t}$ for $z = \arctan(xy)$, $x = t^2$, $y = se^t$

ans $\dfrac{t^2e^t}{1+t^4s^2e^{2t}}$, $\dfrac{ste^t(2+t)}{1+t^4s^2e^{2t}}$

5. (801, #33) The radius of a right circular cylinder is decreasing at a rate of 1.2 cm/s while its height is increasing at a rate of 3 cm/s. At what rate is the volume of the cylinder changing when the radius is 80 cm and the height is 150 cm?

ans $-96 * 10^2\pi$

6 (801, #38) Car A is traveling north on Highway 16 at 90 km/h. Car B is traveling west on Highway 83 at 80 km/h. Each car is approaching the intersection of these highways. How fast is the distance between the cars changing when car A is 0.3 km from the intersection and car B is 0.4 km from the intersection?

ans $-118\ km/h$

7. (801, #39) If z = f(x,y) where $x = r\cos(\theta)$, $y = r\sin(\theta)$ (a) find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ and (b) show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

Check Your Comprehension

1. Let U = U(x,y) be a function of x and y where $x = e^s\cos(t)$, $y = e^s\sin(t)$
Which of the following are true? Which are false? Why?

a. $< x - y, x + y > \bullet < U_x, U_y > = U_s + U_t$

b. $(U_x)^2 + (U_y)^2 = e^{-2s}\left[(U_s)^2 + (U_t)^2\right]$

c. $U_x = e^{-s}\left[\frac{U_s}{\cos(t)} + \frac{U_t}{\sin(t)}\right]$

d. $U_y = e^{-s}\left[\frac{U_s}{\sin(t)} - \frac{U_t}{\cos(t)}\right]$

e. $U_{xx} + U_{yy} = e^{-2s}\left[U_{ss} + U_{tt}\right]$

2. One mole of an ideal gas is contained in a cylinder which has a time dependent volume given by

V(t) = 20 + 5*cos(t) liters

The temperature of the gas is increasing according to

$$T(t) = 300e^{0.1t} \text{ K(elvins)}$$

The pressure, P, of the gas satisfies the relation PV = RT, where R is a constant,

$$R = 0.82$$

How is the pressure of the gas changing at t = 11?

a) Increasing
b) Decreasing
c) Not Changing
d) Can't tell from information given

If the apparatus explodes if the pressure goes above 4 atmospheres, will it still be functioning at $t = 11$?

a) Yes
b) No