

(800,#6) Find dw/dt for $w = \frac{x}{y} + \frac{y}{z}$, $x = \sqrt{t}$, $y = \cos(2t)$, $z = e^{-2t}$

Solution:

$$\frac{dw}{dt} = w_x \frac{dx}{dt} + w_y \frac{dy}{dt} + w_z \frac{dz}{dt} = \frac{1}{y} \frac{1}{2\sqrt{t}} + \left(-\frac{x}{y^2} + \frac{1}{z} \right) (-2\sin(2t)) - \frac{y}{z^2} (-2e^{-2t})$$

$$\frac{\sec(2t)}{2\sqrt{t}} + 2\sin(2t)(\sqrt{t}\sec^2(2t) - e^{2t}) + 2\cos(2t)e^{2t}$$