Sources of information

For the proofs to know for the final:

(a) The ratio test works for series with positive terms. Source: Lecture 2-F, p. 3.

(b) A convergent sequence or series has terms bounded in absolute value. Source: Lecture 4-W, p. 7.

(c) Bessel's inequality holds for piecewise continuous complex-valued periodic functions of period 2π. Source: Lecture 10-M, p. 5.

(d) Be able to explain why complex power series converge in a disk (unless they converge everywhere or just at one point).

What I had in mind for this is not a formal proof but more just to know the following explanation.

For real power series we discussed this fact, which I'll call a theorem:

Theorem: If $\sum_{n=0}^{\infty} c_n x^n$ converges at $x = x_2$ and if $|x_1| < |x_2|$ then the series converges (absolutely) at $x = x_1$.

In other words, if we know the series converges at one point ($x = x_2$) then it also converges at all points closer to the origin, on either side of the origin. So the series converges at $(-x_2, x_2)$ and x_2 itself, at least.

From this we can see that the only possible "set of convergence" is an interval symmetrical about the origin, except possibly for end points. The interval could be $(-\infty, \infty)$ or $[0, 0]$ or $-R$ to R, open or closed at either end.

Now what about complex numbers? Although we didn't do it in detail, the theorem and its proof are *exactly the same* for complex numbers. It might be good to write z instead of x:

Theorem: If $\sum_{n=0}^{\infty} c_n z^n$ converges at $z = z_2$ and if $|z_1| < |z_2|$ then the series converges (absolutely) at $z = z_1$.

In other words, if we know the series converges at one point ($z = z_2$) then it also converges at all points closer to 0 (the origin). These points form a disk centered at the origin. The disk is "open" since its boundary (the circle) is not included. So the series converges at least in this disk and at z_2 itself.

From this we can see that the only possible "set of convergence" for a complex power series is one of these:

(i) just 0; or

(ii) the whole complex plane; or

(iii) a disk of radius R and center 0, possibly with some points on its circular boundary.