Assignment #9

Note **quiz** announcement below.

Homework assignment #9 is due in lecture Friday, June 2. Be sure to try these problems before your discussion section!

section	page	To do but not hand in	To hand in
§6.2	p. 370	1, 3, 11, 13	4, 6, 8, 12, 14
§6.3	p. 374	1, 3, 9, 12, 15	2, 4, 10, 14, 16
§6.4	p. 384	3	
below		Q-1, Q-3	Q-2, Q-4

Quiz in section, Week 9 (May 30 and June 1): Know:

(a) formulas for $\cosh x$, $\cos(x+y)$, $\cos(x-y)$, $\sin(x+y)$, $\sin(x-y)$;

(b) formula for $\cos(mx)\cos(nx)$ and how to derive it;

(c) $\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,dx$ (for $m, n \geq 0$, cases $m = n$ and $m \neq n$);

(d) formulas for the Fourier coefficients a_n, b_n of a periodic function f of period 2π;

(e) the reasoning showing how the formulas in (d) are derived.

Problem Q-1. Let S be any set of real numbers bounded above. As you know, an upper bound for S is any M so that $s \leq M$ for all s in S. It is a fact that S always has a *least* upper bound, called the *supremum* of S, or $\sup S$. If S is not bounded above, we can write $\sup S = \infty$. If S has a maximum element (i.e., largest number), then that is $\sup S$. Find $\sup S$ in the following cases.

(a) S is the interval $(-\infty, 2)$.

(b) S is the set of all numbers $3 - \frac{1}{n}$, $n = 1, 2, 3, \ldots$.

(c) S is the set of all numbers $3 + \frac{1}{n}$, $n = 1, 2, 3, \cdots$.

(d) S is the set of all integers (whole numbers).

(e) S is the set of all values of $\dfrac{x}{x+1}$ for $x \geq 1$.

Problem Q-2. The "sup norm" of a function f on an interval I is the sup of the set of values of $|f|$ on I. We denote the sup of f on I by $\|f\|$. (Often people write $\|f\|_\infty$, but we won't.) If f has a maximum absolute value on I, then that's the value of $\|f\|$. Find $\|f\|$ in these cases:

(a) $f(x) = xe^{-x}$ on $(0, \infty)$.

(b) $f(x) = x^2$ on $[-4, 3]$.

(c) $f(x) = x^2$ on $[0, 2)$.

(d) $f(x) = 1/x$ on $(0, \infty)$.

As you see, this concept is handy because $\|f\|$ always has a value even if there is no actual maximum value. Remember, though, that a continuous function on a closed interval $[a, b]$ does have a maximum value (so that's its sup norm).

Problem Q-3. For functions f, g on an interval, $\|f - g\|$ is essentially the maximum distance between the two functions. Draw a sketch showing two continuous functions f, g on $[0, 2]$ with $\|f - g\| < \frac{1}{4}$, with f above g in some places and below in others.

Problem Q-4. The sup norm gives an easy way of thinking about uniform convergence: $f_n \to f$ on an interval I means that $\|f_n - f\| \to 0$ as a sequence of numbers. That's all! (Let's stay away from cases where the value would be infinite; in such a case we'd have to say that the value of $\|f_n - f\|$ is eventually finite). In any problem on uniform convergence you can write $\|f_n - f\|$ instead of M_n or ϵ_n.

Do p. 163, Problem 5 using this notation.