Math 33B/1 **S00 M.**

Assignment #7

Note **quiz** announcement below.

Homework assignment #7 is due in lecture Friday, May 19. Be sure to try these problems before your discussion section!

section	page	To do but not hand in	To hand in
§3.11	p. 164	21, 23	20, 22 [1]
§3.13	p. 174	1, 3	2, 4
§3.15	p. 195	1, 13	14, 18, 20
below		M-1	M-2, M-3, M-4, M-5, M-6

Quiz in section, Week 7 (May 15 and 17): One problem similar to p. 163, Ex. 5 or the same on [0,1] (both were done in lecture 6-W), or else the proof that if a power series $\sum_{n=0}^{\infty} c_n x^n$ converges at $x = x_2$ and if $|x_1| < |x_2|$ then it converges at $x = x_1$ (lecture 6-F).

Problem M-1. (a) Find the power series expansion of $\cosh x$ about $x = 0$, each of two ways: (i) Taylor/Maclaurin and (ii) your knowledge of the power series for e^x.

(b) Determine the radius of convergence of this power series.

Problem M-2. Here is a well behaved function that has two different formulas: Let $f(x)$ be defined by $f(x) = 1 + \frac{1}{2!}x + \frac{1}{4!}x^2 + \cdots = \sum_{n=0}^{\infty} \frac{1}{(2n)!}x^n$, like the hyperbolic cosine except that all powers of x appear. Here x is real.

(a) Determine the radius of convergence of this power series.

(b) Find a formula for $f(x)$ when $x \geq 0$. (You'll need to use a square root. Your formula should be an algebraic expression perhaps involving familiar "transcendental" functions such as trig functions, exponential functions, etc.; it should not involve an infinite sum or limit.)

(c) Similarly, find a formula for $f(x)$ when $x \leq 0$.

(d) Find the (real) solutions of $f(x) = 0$.

[1] Notice these ask you to check an integral also.

Problem M-3. For complex numbers, prove (a) $\overline{zw} = \bar{z}\bar{w}$;

(b) $|zw| = |z||w|$.

(For (a), write $z = a + bi, w = c + di$, etc.; for (b) use (a) somehow.)

Problem M-4. (a) Find the sum $1+z+z^2+\cdots$ in the case $z = \frac{1}{2}i$ three ways: (i) by using the formula for the sum of a geometric series, (ii) by collecting the real and imaginary terms together separately and summing the two real series that you get, and (iii) approximately graphically by hand-plotting the partial sums through the 4th power.

(b) See if your answer checks with the on-line demo on summing a complex geometric series.

(c) Describe what happens when you take z outside the region of convergence in the on-line demo.

Problem M-5. (a) Find $\lim_{n\to\infty}(1 - \frac{1}{n})^n$.

(b) If you start with \$100 and take 99% of it, then 99% of that, etc., a hundred times, about how much do you have left?

(c) Suppose you do an activity repeatedly with a $\frac{1}{N}$ chance of success, as in Problem J-1(b), where N is some fixed integer. You saw that the expected number of tries to get a success is N. What is the probability that the first success comes by the N-th try, approximately? (Method: It's 1 minus the probability of no successes in N tries.)

(d) A daredevil does a stunt with a chance of injury of 2%. What are his chances of performing this stunt on 50 occasions with no injury, approximately?

(e) On a busy street near campus, with one lane in each direction, at an intersection it seems that one car in ten waits to make a left turn, blocking traffic. If you find that there are ten cars ahead of you as you come to the intersection, what is the chance that you'll be blocked, approximately?

(Note. In these problems, obviously how good the approximation is depends on N, but it's not bad even for small N.)

Problem M-6. Suppose $\sum_{n=0}^{\infty} c_n x^n$ is a power series for which $\lim_{n\to\infty} |c_n|$ has a limit, say L. By using the ratio test, verify that this series, its derivative, and its integral (with any choice of constant of integration) all have the same radius of convergence. (Do just the case $L \neq 0$.)