Assignment #6

No quiz during Week 6

This assignment is due in lecture Friday, May 12. Be sure to try these problems before your discussion section, even if you have to get some information from reading the text.

section	page	To do but not hand in	To hand in
§3.1	p. 96	1, 9, 13, 17, 25, 31, 37	2, 10, 14, 18, 26, 34, 36
§3.11	p. 163	1, 3, 5, 7, 11	2, 4, 6, 8, 12
below		L-1	L-2, L-3

Problem L-1. (a) Invent an example of functions $f(x)$ and $g(x)$ so that $\lim_{x\to 0} f(x) = 0$, $\lim_{x\to 0} g(x) = 0$, and $\lim_{x\to 0} f(x)^{g(x)} = 0$;

(b) Invent an example of functions $f(x)$ and $g(x)$ so that $\lim_{x\to 0} f(x) = 0$, $\lim_{x\to 0} g(x) = 0$, and $\lim_{x\to 0} f(x)^{g(x)} = 1$;

Problem L-2. How do textbook writers invent l'Hôpital problems? One way is to use power series, which students don't usually know about when they study l'Hôpital. But you do.

Invent three interesting l'Hôpital problems using your knowledge of power series. In your problems you should take advantage of the expansions of e^{x^2}, $\log(1+x)$, and $\sin x$, either alone or combined. (Use all three, not necessarily together.) Your problems should not be identical to examples done in lecture.

(Sample: In lecture we'll probably do the example $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$. To invent it, think this way: $\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots$, so $1 - \cos x$ starts with an x^2 term, so dividing by x^2 will give a power series starting with a nonzero constant (in fact, $\frac{1}{2}$). Then $\lim_{x\to 0} \dfrac{1-\cos x}{x^2} = \frac{1}{2}$, which makes an interesting l'Hôpital exercise requiring two steps.)

(Or we could make a variation by using the expansion of $\cos 3x$, getting the problem $\lim_{x\to 0} \dfrac{1-\cos 3x}{x^2}$. Or we could square the fraction, to make the problem $\lim_{x\to 0} \dfrac{(1-\cos x)^2}{x^4}$. Or we could turn it upside down, getting the problem $\lim_{x\to 0} \dfrac{x^2}{1-\cos x}$.)

Problem L-3. For the following limits, using a calculator as needed, find (1) the limit, (2) the value of the expression at 10 or .1, depending on whether the limit has $n \to \infty$ or $x \to 0$, and (3) at 100 or .01, similarly.

(a) $\lim_{x\to 0} \dfrac{x^2 - 3x + 2}{x^2 - 4x + 5}$

(b) $\lim_{x\to 0} \dfrac{\sin x}{x}$

(c) $\lim_{x\to 0} x^x$

(d) $\lim_{n\to\infty} n^{1/n}$

(e) $\lim_{n\to\infty} (1 - \frac{1}{n})^n$

(f) $\lim_{n\to\infty} \dfrac{\log x}{x}$

(We could use x for n; it's just that n is more familiar from sequence and series problems.)