Assignment #5

No quiz during Week 5, because of the midterm.

This short assignment is due in lecture Friday, May 5. Be sure to try these problems before your discussion section, even if you have to get some information from reading the text.

section	page	To do but not hand in	To hand in
§3.9	p. 146	8, 9, 13	10, 12
§3.10	p. 153	3, 13, 21, 23	2, 4, 16, 22
below			J-1

Problem J-1. As mentioned in lecture, if you have some game or experiment where you have different probabilities of getting different scores, the "expected value" is the sum of the scores weighted by the probabilities—a weighted average. Examples:

1. Suppose you have probability $\frac{1}{6}$ of getting 7, probability $\frac{1}{3}$ of getting 13, and probability $\frac{1}{2}$ of getting 2. (That's all, since $\frac{1}{6}+\frac{1}{3}+\frac{1}{2}=1$.) Then the expected value is $\frac{1}{6}\cdot 7+\frac{1}{3}\cdot 13+\frac{1}{2}\cdot 2=\frac{39}{6}=6.5$.

2. In a lottery, if you have a chance of one in a million of winning \$2 million (and otherwise you will win nothing), then your expected value is $0.000001\cdot 2000000+0.999999\cdot 0=\$2$ (so you'd better not have to pay much more than that for a ticket).

3. If you throw a die repeatedly until you get a 5, the expected number of throws is $\frac{1}{6}\cdot 1+\frac{5}{6}\cdot\frac{1}{6}\cdot 2+\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}\cdot 3+\cdots=\sum_{n=1}^{\infty}(\frac{5}{6})^{n-1}\frac{1}{6}n$, because your chance of needing one throw is $\frac{1}{6}$, your chance of needing two throws is $\frac{5}{6}\cdot\frac{1}{6}$ (one failure followed by one success), and so on. In this example there are infinitely many possibilities, but the same principle applies.

The problem:

(a) In Example 3 above, find an explicit value for the expected number of throws, by adding up the infinite series. (Take the $\frac{1}{6}$ outside and put x for $\frac{5}{6}$ temporarily so it's easier to work with. Is your answer reasonable?)

(b) Suppose you do some other activity repeatedly until you succeed, and suppose the chance of success on each try is $\frac{1}{N}$. Evaluate the same kind of sum to find the expected number of tries, in terms of N.

(c) Although we won't go into it, in Example 3 the sum $\sum_{n=1}^{\infty}(\frac{5}{6})^{n-1}\frac{1}{6}n^2$ is also useful. Find its value.