

Examples

1. Infinite sequences

- (1) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \rightarrow 0$
- (2) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \rightarrow 0$
- (3) $1, r, r^2, r^3, \dots \rightarrow ?$ or more generally
- (3') $a, ar, ar^2, ar^3, \dots \rightarrow ?$ (*infinite geometric progression*)
- (4) Let $a_n = \frac{n^2 + 2n + 3}{3n^2 + n - 1}$. What happens as $n \rightarrow \infty$?
- (5) Let $a_n = \sin n$. What happens as $n \rightarrow \infty$?
- (6) Let $a_n = \frac{1}{n} \sin n$. What happens as $n \rightarrow \infty$?
- (7) If $a_1 = 1$ and $a_{n+1} = \frac{1}{2} \left(a_n + \frac{5}{a_n} \right)$, then $a_n \rightarrow \sqrt{5}$.

See values later on in this handout.

2. Infinite series of numbers

There are some surprises.

- (8) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$
- (9) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots = \frac{2}{3}$
- (10) $1 + r + r^2 + r^3 + \dots = \frac{1}{1-r}$ if $|r| < 1$ or more generally
- (10') $a + ar + ar^2 + ar^3 + \dots = \frac{a}{1-r}$ if $|r| < 1$ (*geometric series*)
- (11) $0.111\dots = \frac{1}{9}$
- (12) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots = \infty$ (*harmonic series*)
- (13) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots = \log 2$ (*alternating harmonic series*)
- (14) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$
- (15) $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots = e$
- (16) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$

