Assignment #5

Problem G-1. Let $X_n = \{1, \dots, n\}$ (a) If you choose a random permutation on X_n, what is the chance that it is a derangement, approximately?

(b) If you choose a random function on X_n to itself, what is the chance that it has no fixed points, approximately? (Here "approximately" means to take a limit as n gets large.)

Problem G-2. Show that derangements obey this recursion:
$D_{n+1} = (n+1)D_n + (-1)^{n+1}$. (Use our original recursion.)

Problem G-3. In the complex plane, given n consider the $2n$-th roots of unity. Join them in pairs with n chords so that no two chords cross. How many ways are there of doing this, in terms of n? (Two ways that are the same under a symmetry are to be counted as different. Answer with an expression in closed form, simplified to the extent that you know how.)

Problem G-4. Find and prove an expression for the n-th partial sum of the sequence of Fibonacci numbers, in terms of Fibonacci numbers.

Problem G-5. Suppose that a codeword on symbols $\{0, 1, 2\}$ is legal if and only if it does not have two consecutive 0's. Let a_n be the number of legal codewords of length n. Use a recurrence to find a closed-form expression for a_n, and describe the asymptotic behavior of a_n.

Problem G-6. Divide the plane into regions by n lines in general position. Find a formula for the number of regions. (Suggestion: Think in terms of a difference table and use Newton interpolation, as described in class.)