Math 206A **W01 F.**

Assignment #4

Due Friday, February 2.

Problem F-1. Let P be a partially ordered set in which every chain is finite and every antichain is finite. Show that P must be finite. (Suggestion: Use Ramsey with $c = 2, r = 2$. What are the colors for pairs?)

Problem F-2. Try to find an intuitive explanation of why the Catalan tree and parenthesization problems have the same generating function, by showing a correspondence between trees and parenthesizations. (At the moment I don't see it myself. If you don't see it either, just write what your approach was.)

Problem F-3. What estimate does Stirling's formula give for $\binom{2n}{n}$?

Problem F-4. Decide whether a fair pair of dice is the same thing as a pair of fair dice.

In other words, if you have a fair pair of six-sided dice then you have a certain chance of throwing each total T from 2 to 12, e.g., $\mathrm{Prob}(T = 2) = \frac{1}{36}$. Suppose you have two "unfair" dice. Is it conceivable that the probability of each total is the same as for the fair dice?

Method: Express the probabilities of totals using generating functions. Relate the generating functions for the totals to generating functions for the individual dice.

Problem F-5. Consider nonempty rooted trees with 0, 1, or 2 branches at each node, where the order of branches matters but the single branch in the 1-branch case is not identified as being left or right. Find the number of such trees with 40 nodes.

Method: Consider the generating function $g(x)$ and find a recursion of the form $g(x) = \cdots$. It will be quadratic. In the following, I'll use the different example $g(x) = x/(1-g(x))$ discussed in class. Start Mathematica on `oak` or a similar server, by using the command `mathematica &` . In Mathematica, a function uses the notation `g[x]`. Type

`solns = Solve[g[x]==x/(1-g[x]), g[x]]` (and press RETURN while holding down the SHIFT key). This says to solve the given equation for `g[x]` and call the answer `solns`. The answer will be a list of two possible solutions, in the form of substitutions `g[x] -> an expression in x` . Consider the first one, denoted in Mathematica by `solns[[1]]` . It could be used to substitute the expression into any other expression involving `g[x]` by using the Mathematica substitution operator `/.`, but just substitute it into `g[x]` itself to end up with the expression alone, as follows:

`expr = g[x] /. solns[[1]]` (SHIFT-RETURN).

Now expand the expression in a Taylor series to degree 40, by

`Series[expr, {x, 0, 40}]` (SHIFT-RETURN), which means to expand using the variable x about $x = 0$ to degree 40.

If the coefficients are all nonnegative, fine; the last one is your answer. If some are negative, go back and try the second solution for $g(x)$.

Notes: (1) Multiplication in Mathematica is indicated by `*`, or simply by juxtaposition with a space between: `a*b` or `a b` . The space is necessary because `ab` is a legal name for a variable. (2) If you mistype something you can go back and edit in the standard way. (3) If you are accessing the system from home via telnet then you won't be able to use this graphical interface; instead, type `math` to get a line-by-line version of Mathematica, and in running Mathematica use plain RETURN instead of SHIFT-RETURN. (4) We have only a certain number of Mathematica "keys", so if several other people are using Mathematica you may not be able to run it. (5) If Mathematica doesn't know a detailed expansion for some quantity then it will just repeat the quantity back. This could happen, for example, if you mistype `solns[[1]]` as `sols[[1]]` . (6) It isn't really necessary to give a name to each line, since `%13` means the output of line 13, etc. (7) Maple has similar capabilities. (8) Ask me for help if you need it.