Assignment #2

Due Friday, January 19[1].

Problem C-1. The six faces of a cube are to be painted with six different colors, each face with a distinct color. Use Burnside's Theorem to determine in how many ways this be done:

(a) if colorings that differ by a rotation are counted as different;

(b) if they are counted as the same.

Problem C-2. In how many ways-can the six vertices of a regular hexagon be colored with at most two colors?

(a) if colorings that differ by a rotation are counted as different?

(b) if they are counted as the same?

Problem C-3. A rod divided into six segments is to be colored with one or more of n different colors.

(a) Use Polya's Method to determine in how many ways this can be done.

(b) Same question, but for a seven-segment rod.

Problem C-4. Suppose that the groups G and H act, respectively, on the sets D and R. Let $F = \{f : D \to R\}$. We say that $f_1 \in F$ is related to $f_2 \in F$ if there exist $g \in G$ and $h \in H$ such that $\pi_h(f_1(d)) = f_2(\pi_g(d))$ for all $d \in D$. Prove that this is an equivalence relation.

Problem C-5. The sides of a square are colored using three colors. Two colorings are said to be equivalent if one can be obtained from the other by a rotation of the square and/or by a permutation of the colors. Use de Bruijn's Method to find the number of equivalence classes of coloring schemes.

[1]problems selected by Michael D. Miller based on his lectures.