Assignment #1

Due Friday, January 12.

1. Give an intuitive explanation of this identity:

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

2. Let's agree that any two people can be said to be friends or not. Show that in any group of six people there are three each two of whom are friends or three each two of whom are not friends.

3. Show that a set of mutually orthogonal $n \times n$ Latin squares cannot have more than $n-1$ members. (Suggestion: standardize the squares somewhat by making the strongest assumption you can on their first rows, without loss of generality. Then look at some particular entry position not in the first row.)

4. Do (a) or (b).

(a) Show that for any list of 13 integers, there is a nonempty run of consecutive terms whose sum is divisible by 13.

(b) Let P be the set of all people who have ever lived. Every member of P has shaken hands with other members of P some number of times. Show that the number of members of P who have shaken hands with other members of P an odd number of times is even.