Math 149 **S03 R.**

Sample midterm

(In an actual version, problems would be on separate pages with space for answers. Each problem counts 10 points.)

1. Give a map that is a rotation of $\mathbf{E}^2$ by $90°$ *clockwise* with center $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Your answer may include one or more matrices, but with explicit numeric entries.

2. Sketch a cubic parametric curve $Q(t)$ such that $Q(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $Q'(0) = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$, $Q(1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $Q'(1) = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$. Indicate axes and label some points on the axes to indicate the scale. Take care to plot $Q(\frac{1}{2})$ exactly.

3. Suppose $f(t, u)$ is a biquadratic polynomial function that is 0 at all the points (i, j) for $i = 0, 1, 2$ and $j = 0, 1, 2$, except that $f(1, 1) = 1$. Find $f(\frac{1}{2}, \frac{1}{2})$. (There is only one function that fits this description, so if you can think of one, that's it. A biquadratic polynomial is quadratic in each variable when the other is held fixed.)

4. For the parametric bilinear patch with data points P_{ij}, $i = 0, 1$, $j = 0, 1$, find a normal vector at the point where $t = 0$, $u = 0$.

5. Short-answer questions:

(a) Sketch the points that are convex combinations of the vertices of the standard triangle. (A convex combination is a barycentric linear combination with nonnegative coefficients.)

(b) Draw a Bézier control polygon with four control points at random and indicate graphically how to use the de Casteljau method to calculate $P(\frac{1}{4})$.

(c) For the cubic Bézier curve $P(t)$ in $\mathbf{R}^2$ with control points $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 10 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 20 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, find $P''(0)$ explicitly.

(d) Invent an example of a ruled parametric surface (a surface that can be swept out by a moving straight line). Your answer should be some kind of explicit formula for a function $P(t, u)$.