Assignment #1

Announcement: One of the other faculty members will substitute on Monday, April 7, since I'll be out of town. For the same reason, I will not have office hours on Monday or Tuesday. Instead, I'll have an extra office hour **Thursday, April 10, 2:30-3:30** (changed from a previous version).

The homework was supposed to be due on Wednesday, April 9, but this time it's OK to hand it in **Friday, April 11** instead. Do look over the problems before the discussion section on Tuesday so that you can ask questions!

Reading: Chapters 1, 3. Look through Chapter 2 lightly; we may come back to this later.

To do but not hand in:

p. 11, Ex. 2;

C-1, C-2, C-4 [meaning from the Problems part of Handout C]

D-1 (below—do try this!)

To hand in:

p. 11, Ex. 5;

C-3;

D-2, D-3, D-4, D-5.

Problem D-1. Does the vector $(1,1,1)$ make a $45°$ angle with the z-axis?

Problem D-2. (a) Let $M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, a reflection in the x-axis. Show that $MR_\theta M = R_{-\theta}$, both by a calculation and by describing what the effect is if you do the left side to an object such as a piece of paper. (For clarity, imagine using paper with printing on it.)

(b) Show that $R_{-\theta}M = MR_\theta$. (Use (a), or calculate directly.)

(c) Show that the matrix of a reflection in $\mathbf{R}^2$ whose mirror line makes an angle θ with the x-axis is $R_{2\theta}M$. (Start from the "three-step method.")

Problem D-3. There are only two 2×2 rotation matrices that have real eigenvalues. (a) Which ones? (b) Why are there no others? (Give some explanation.)

Problem D-4. Express a rotation of 45° counterclockwise in $\mathbf{E}^2$ about the center $(1,1)$. Your answer should involve only one constant matrix and (if needed) one constant vector, with explicit entries. (Start by rewriting $(1,1)$ as a column vector. An explicit entry can be an expression, such as $\frac{1}{2}+\frac{1}{2}\sqrt{3}$. Leave in mathematical form rather than decimal approximations.)

Problem D-5. For the affine map $T : \mathbf{E}^n \to \mathbf{E}^n$ given by $T(\mathbf{p}) = A\mathbf{p} + \mathbf{v}$, with A nonsingular, show that the inverse map T^{-1} is also affine and say explicitly what its linear and translational parts are in terms of A and $\mathbf{v}$.