Assignment #8

Quiz 8 in discussion section **Tuesday, November 20**:

There will be no Quiz 8, since it would be the day after the midterm.

Assignment due nominally in lecture on Wednesday, November 21 but you can hand it in a week late on **Wednesday, November 28**. However, please look at the material earlier or you will have more and more trouble following the lectures and also it will get in the way of Assignment #9.

where	Do but don't hand in	Hand in
U	U-2, U-3, U-4, U-5 U-6, U-9, U-12	U-8, U-10, U-12, U-15
V	V-19, V-20	V-16, V-17, V-18
Z	Z-1	Z-2

Problem Z-1. As mentioned in Problem Q-1(g), the negation of "for all x, the property P holds" is "there exists x for which p doesn't hold" (where P is some statement mentioning x). In other words, if we write $\neg$ for "not",

$\neg(\forall x)(P)$ is the same as $(\exists x)(\neg P)$. Notice that the "not" is moved inside and "for all" becomes "there exists" Similarly,

$\neg(\exists x)(P)$ is the same as $(\forall x)(\neg P)$.

The set $\mathbb{R}$ of real numbers has the property that between any two numbers there is another one. In other words, $\mathbb{R}$ has the property

$(\forall x)(\forall y > x)(\exists z)(x < y \text{ and } y < z)$.

The set $\mathbb{Z}$ of integers does not have this property. Write down a logical statement that says what holds for $\mathbb{Z}$, putting "not" as far inside as possible.

Notes.

(1) "not (P and Q)" is the same as "(not P) or (not Q)".

(2) It's a little informal to put ">" in with the quantifiers, as in "$(\forall y > x)$", but it's often done and is easier for our purposes. In doing a negation, the ">" doesn't change.

Problem Z-2. To do definitions and proofs in calculus is actually harder than for linear algebra, partly because their logical statements are more complicated, going back and forth between "for all" and "there exists". For example, "f is a continuous function" is expressed by the statement, "for all x and for all $\epsilon > 0$, no matter how small, there is a $\delta > 0$ small enough so that for all y, if $|x - y| < \delta$ then $|f(x) - f(y)| < \epsilon$".

In shorter notation, without the explanatory part about big and small, this says

$$(\forall x)(\forall \epsilon > 0)(\exists y)(|x - y| < \delta \Rightarrow |f(x) - f(y) < \epsilon).$$

If f is not continuous, what logical statement holds? Put "not" as far inside as possible. (The negative of $P \Rightarrow Q$ is P but not Q, i.e., P and $\neg Q$.)

Notes.

(1) In logic, the symbols $(\forall)$ and $(\exists)$ are called "quantifiers".

(2) Going back and forth between $(\forall)$ and $(\exists)$ is called "alternation of quantifiers".