Assignment #1

Quiz 1-T, October 2. Be able to find the general solution to a set of linear equations, using row reduction—even if the system is not square or is square but singular.

Read/Review Chapter 1, handouts B and D, and start Chapter 2.

Homework due in lecture **Wednesday, October 3**.

In this course, you should always give your reasoning, even if a problem doesn't explicitly say to.

where	Do but don't hand in	Hand in
p. 5	5, 7	
p. 33	1, 5	
p. 39	1, 4, 5	2, 3
B	B-3, B-5	B-1, B-2, B-4
C	C-5	C-1, C-2, C-3, C-4
D	D-1	D-2

Problem C-1. Consider the set $S = \{1, 2, 3, 4\}$. Starting from the subsets $\{1, 2\}$ and $\{1, 3\}$ and using the set operations of union $\cup$, intersection $\cap$, and complementation $'$ (repeatedly), how many different subsets of S can you make? (Give your reasoning.)

Problem C-2. You find a torn page from an instructor's gradebook, with the data shown below. The last average is torn off. What is it? (These are weighted averages.)

labs	hmwk	midt	final	avg
12	24	25	25	23
50	30	26	36	35
60	60	75	100	85
40	80	50	75	63
60	80	100	100	

Problem C-3. Prove that for any prime number p, $\sqrt{p} \notin \mathbf{Q}$. (Use any reasonable facts about numbers that do or do not have p in their prime factorizations.)

Problem C-4. Show that in the definition of a vector space, the law $1v = v$ cannot be proved from the other laws. (You may assume that the vector space is over $\mathbb{R}$.)

(Method: Make an example of a set V with an operation $+$ and multiplication by scalars in $\mathbb{R}$ for which $1v = v$ fails but all other defining laws for a vector space are true.

Suggested way: Take $\mathbb{R}^2$ and fudge it by using $+$ as usual but declaring that multiplication by scalars always gives the result $\mathbf{0}$. Of course, to have a good list of laws for a definition, no law listed should be provable from all other laws. This problem verifies just one instance of this goal.)

Problem C-5. (a) What is the subspace of $\mathbb{R}^3$ spanned by $(1, 0, 0)$, $(2, 3, 0)$, and $(4, 5, 6)$? (Give a simpler description.)

(b) What is the subspace of the set $\mathrm{Pols}(\mathbb{R}, 2)$ of degree at most 2 that is spanned by 1, $2 + 3x$, and $4 + 5x + 6x^2$?

(c) What is the subspace of the $\mathrm{Pols}(\mathbb{R}, 2)$ spanned by x^2, $2x^2 + 3x$, and $4x^2 + 5x + 6$?