Assignment #10

Due in lecture **Friday, June 12 (not Wednesday)**

Announcements:

Pedro will hold a review session this Monday, June 8, 4:00-6:00 in MS 3970.

In Week 10 the Tuesday and Friday sections will be switched. I'll lecture Monday, Tuesday, Wednesday, and Pedro will hold a discussion section on Friday.

Problems to do but not hand in:

Do the problem in Handout T that were not previously assigned: T-10, T-11 (which means diagonalizable using complex numbers), T-12, T-13, T-14, T-15.

Problems to hand in:

Do these:

Problem DD-1. Use Gram-Schmidt to construct orthonormal bases in these cases:

(a) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$

(b) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$

Problem DD-2. Diagonalize A with a rotation matrix in these cases:

(a) $A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix},$

(b) $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$

(continued)

Problem DD-3. (a) Show that if the eigenvalues of a matrix A are all the same (for example, the eigenvalues are 5,5,5 if A is 3×3, so that the characteristic polynomial is $-(\lambda - 5)^3$), but A is not a scalar matrix, then A is not diagonalizable at all.

(Easy if you use Problem AA-1.)

(b) Give a 3×3 example of a nonsingular matrix that is not diagonalizable.

Problem DD-4. (a) Show that if A is similar to B using P and $p(x)$ is a polynomial, then $p(A)$ is similar to $p(B)$. (For example, $A^2 - 3A + I$ is similar to $B^2 - 3B + I$.)

The Cayley-Hamilton theorem says that any square matrix A acts like a root of its own characteristic polynomial. In other words, if $p_A(\lambda)$ is the characteristic polynomial of A, then $p_A(A)$ is the zero matrix.

(b) Try this for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(c) Prove it when A is any 2×2 matrix by writing $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(d) Prove it when A is any $n \times n$ diagonalizable matrix, by using (a).

Problem DD-5. Prove that if A is similar to B then A and B have the same characteristic polynomial. (Use the definition of the characteristic polynomial and put in P, P^-1.)