Math 113 **F99 R.**

Assignment #6

Problems due in lecture on **Friday, November 12**:

§	page	NTHI	THI
§4.3	p. 170	1(b,c,g)[1], 6	1(d,f,g,h)[1], 3, 7
§4.4	p. 175	1, 5	8, 9
§4.5	p. 183	1, 6(b,c,f)[1], 8	2(b,d), 6(e,g,h)[1], 7
below		R-1, R-2, R-3	R-4, R-5, R-6, R-7

Problem R-1. In lecture, we found that choosing alphabet cookies from a pile of 5 and a pile of 7 gives the same generating function $(1+x)^{12}$ as choosing from (a) one pile of 12 and (b) twelve piles of one. Explain without generating functions why this is to be expected.

Problem R-2. Using a scientific calculator, find the numerical values of the first four terms in the series expansion of $(1.1)^{-\frac{1}{2}}$, i.e., of $(1+x)^{-\frac{1}{2}}$ when $x = 0.1$. Add these terms and check the total against the value of $\frac{1}{\sqrt{1.1}}$ found by using the square root function on the calculator.

Problem R-3. Suppose we draw a circle and split it into regions by drawing lines. Let a_k be the maximum number of regions you can get using k lines.

(a) Write down the values of a_0, a_1, a_2.

(b) Make a conjecture about a_k, based on (a).

(c) Is this conjecture correct?

(d) What happens for the analogous problem of splitting a sphere into solid regions by using k planes?

(This problem shows the dangers of informal reasoning simply saying "and so on" instead of using an actual proof.)

Problem R-4. Make a conjecture about the sum of the first n odd integers (starting with 1), write it in sigma notation, and prove it using formal induction.

Problem R-5. Make up a problem on rook polynomials whose answer is the generating function $(1+3x+x^2)^2$. (An example done in lecture may help.)

[1] Notice that you are not asked to calculate a numerical answer.

Problem R-6. This is a strange problem but it illustrates how general the use of generating functions is.

You are given n tokens. You may use as many of your tokens as you want to take some alphabet cookies from a pile of 7 alphabet cookies A-G (as in lecture), giving one token for one cookie. You may use as many as you want of the rest of your tokens to choose some cookies from an infinite pile of indistinguishable cookies (so there's exactly one way of doing that). You must use any remaining tokens to place rooks on a 3 by 3 board (with all nine squares eligible, and with no two rooks in the same row or column). If the number of ways of using the n tokens is c_n, find an expression for the generating function of the sequence $c_0, c_1, c_2, \ldots$. (Your answer should be in closed form, i.e., an algebraic expression not involving an infinite sum.)

Problem R-7. In a graph G, a subset S of the set of vertices is *independent* if no two vertices in S are neighbors (i.e., no two are joined by an edge).

(a) Using the concept of independent subsets, explain what it means for G to be n-colorable. (In other words, re-express the idea of being n-colorable using new terminology.)

(b) Let a_k be the number of different k-element independent subsets of G. Thus we have a sequence $a_0, a_1, \cdots$ and so we get a generating function, which we can call the "independence generating function". Find the independence generating function of G (i) if G is a 4-cycle, (ii) if G is a 5-cycle, and (iii) if G is a 6-cycle. (In each case, G consists of a cycle with no other edges.)

(c) Find the independence generating function for a graph that is the union of three separate connected components: a 4-cycle, a 5-cycle, and a 6-cycle. (Use (b) somehow.)