# MATH 32B(1) MIDTERM TWO

Name ____________________________
Student ID# ____________________________
discussion Section ____________________________

1_________,2_________, 3_________,4_________,5_________,Total score _________

For problems 1,2,3,5(a), you only need to give answers.
For problems 4,5(b), you should show your work in detail.

1. (18 points) Let $E$ be the region of integration for the integral

$$\int_0^1 \int_0^{x^3} \int_0^y f(x,y,z)dzdydx$$

Then

(a) $E = \underline{\{(x,y,z)|0 \le x \le 1, 0 \le y \le x^3, 0 \le z \le y\}}$.

(b)

$$\iiint_E f(x,y,z)dV = \int_{\underline{0}}^{\underline{1}} \int_{\underline{0}}^{\underline{x^3}} \int_{\underline{z}}^{\underline{x^3}} f(x,y,z)dydzdx$$

(c)

$$\iiint_E f(x,y,z)dV = \int_{\underline{0}}^{\underline{1}} \int_{\underline{z}}^{\underline{1}} \int_{\underline{y^{\frac{1}{3}}}}^{\underline{1}} f(x,y,z)dxdydz$$

2. (12 points) Find the potential function $f$ of the vector field

$$\vec{F} = (1 + y\sin(xy))\vec{i} + (y^2 + x\sin(xy))\vec{j}$$

$f = \underline{-\cos(xy) + x + \frac{1}{3}y^3 + K}$.

3. (20 points) Determine whether the following statements are true or false:

(a) $\underline{F}$ $\vec{F}(x,y) = \sin(xy)\vec{i} + \cos(x)\vec{j}$ is conservative on $\mathbb{R}^2$.

(b) $\underline{T}$ $\vec{F}(x,y) = (e^x + \sin(x+y) + y^2)\vec{i} + (\sin(x+y) + 2xy + 2)\vec{j}$ is conservative on $\mathbb{R}^2$.

(c) $\underline{F}$ The region $D = \{(r,\theta)|r > 0\}$ represented by polar coordinates in the xy-plane is open simple-connected.

(d) $\underline{T}$ Let $C$ be an oriented curve in $\mathbb{R}^2$, then for general function $f(x,y)$, we have $\int_{-C} f(x,y)ds = \int_C f(x,y)ds$.

(e) $\underline{F}$ Let $C$ be an oriented curve in $\mathbb{R}^2$, then for general function $f(x,y)$, we have $\int_{-C} f(x,y)dx = \int_C f(x,y)dx$.

4. (25 points)Use Green's theorem to find the work done by the force

$$\vec{F}(x, y) = (-y + e^{\sqrt{x}})\vec{i} + (x + \cos y^2)\vec{j}$$

in moving a particle counterclockwise from $(1, 0)$ to $(-1, 0)$ along the unit circle $x^2 + y^2 = 1$, then back to the point $(1, 0)$ along x-axis.(write down each step in detail)

**Solution:** The oriented closed curve is the boundary of

$$D = \{(x, y)|x^2 + y^2 < 1, x > 0\}$$

.

$$\int_C \vec{F} \cdot d\vec{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA$$

$$= \iint_D \frac{\partial(x + \cos y^2)}{\partial x} - \frac{\partial(-y + e^{\sqrt{x}})}{\partial y} dA = \iint_D 2dA = \pi.$$

The last equation is because $D$ is the half of the unit disk.

5. (25 points) let $E$ be the solid bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ in the space $\mathbb{R}^3$.

(a) Use triple integral to represent the volume of $E$ (you are not required to evaluate the intergral).

(b) Evaluate the triple integral (write down each step in detail)

$$\iiint_E \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}dV$$

**Solution**

(a) the volume of $E$ is $\iiint_E 1dV$

(b) Let $T$ be the transformation given by $x=au, y=bv, z=cw$. The the determination of the Jocobian is

$$|\frac{\partial(x,y,z)}{\partial(u,v,w)}|=abc.$$

Let $D$ be the unit ball $\{(u,v,w)|u^2+v^2+w^2<1\}$ then $T$ transform $D$ to $E$. So

$$\iiint_E \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}dV$$

$$=\iiint_D (u^2+v^2+w^2)|\frac{\partial(x,y,z)}{\partial(u,v,w)}|dV$$

$$=\iiint_D (u^2+v^2+w^2)abcdV$$

We use spherical coordinators

$$=abc\int_0^{2\pi}\int_0^{\pi}\int_0^{1}\rho^4\sin\phi d\rho d\phi d\theta$$

$$=abc\int_0^{2\pi}\int_0^{\pi}\frac{1}{5}\sin\phi d\phi d\theta=\frac{4}{5}\pi abc$$