**Quiz 4, Monday, June 1**

Let

$$A = \begin{pmatrix} 4 & 1 & -1 \\ 1 & 7 & 2 \\ -1 & -1 & 4 \end{pmatrix}.$$

Find the general solution to $y' = Ay$. The characteristic polynomial of $A$ is $(\lambda - 3)(\lambda - 6)^2$. $A$ has eigenvectors $(2, -1, 1)^T$ and $(1, 1, -1)^T$; the vector $(-1, 2, 1)^T$ may be useful to you.

Solution: Call $(2, -1, 1)^T$ $v_1$,$(1, 1, -1)^T$ $v_2$, and $(-1, 2, 1)^T$ $v_3$. We have solutions $y_1 = e^{3t}v_1$, $y_2 = e^{6t}v_2$, and since $(A - 6I)^2 v_3 = 0$,

$$\begin{aligned} y_3 = e^{tA}v_3 = e^{t6I+t(A-6I)}v_3 &= e^{t6I}e^{t(A-6I)}v_3 \\ &= e^{6t}\left[v_3 + t(A - 6I)v_3\right] \\ &= e^{6t}\left[v_3 + 3tv_2\right] \end{aligned}$$

is a third fundamental solution. The general solution is then $C_1y_1 + C_2y_2 + C_3y_3$.