Quiz 3, Friday, May 15

Let

$$A = \begin{pmatrix} 0 & 4 \\ -2 & -4 \end{pmatrix}.$$

Find the general solution to $y' = Ay$. For full credit, write your solution without using any complex numbers.

Solution: the characteristic polynomial of A is $\lambda^2 + 4\lambda + 8$, with roots $\lambda_1 = -2 + 2i$ and $\lambda_2 = -2 - 2i$, and eigenvectors

$$v_1 = \begin{pmatrix} 2 \\ -1+i \end{pmatrix}$$

and

$$v_2 = \begin{pmatrix} 2 \\ -1-i \end{pmatrix}.$$

One of the complex valued fundamental solution to the equation is given by $y_{\text{fc}} = e^{\lambda_1 t} v_1$. The real fundamental solutions are given by $y_1 = \text{Re}(y_{\text{fc}})$ and $y_2 = \text{Im}(y_{\text{fc}})$.

$$y_{\text{fc}} = e^{-2t}(\cos 2t + i \sin 2t) \begin{pmatrix} 2 \\ -1+i \end{pmatrix}$$

$$= e^{-2t}\left[\cos 2t \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \sin 2t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + i\cos 2t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + i \sin 2t \begin{pmatrix} 2 \\ -1 \end{pmatrix}\right].$$

So

$$y_1 = e^{-2t} \begin{pmatrix} 2\cos 2t \\ -\cos 2t - \sin 2t \end{pmatrix}$$

and

$$y_2 = e^{-2t} \begin{pmatrix} 2\sin 2t \\ \cos 2t - \sin 2t \end{pmatrix}.$$

The general solution is $C_1 y_1 + C_2 y_2$.