Quiz 2 solutions

i) Find a solution to the initial value problem

$$y'' - 2y' + 2y = 0,$$
$$y(0) = 1,$$
$$y'(0) = 0.$$

Is it possible that there is more than one solution to the initial value problem?

Solution: the characteristic polynomial of the equation is $x^2 - 2x + 2$ with roots at $1+i$ and $1-i$. Thus the general solution to the equation is given by $e^t(C_1 \cos t + C_2 \sin t)$. Plugging in $y(0) = 1$ and $y'(0) = 0$ gives the system

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

and so $C_1 = 1$ and $C_2 = -1$; so $y(t) = e^t \cos t - e^t \sin t$.

It is not possible that there is more than one solution to the initial value problem by theorem 1.17 in the book- the functions p, q, and g in this case are constants, and so in particular are continuous.

ii) Find a solution to the initial value problem

$$y'' - 4y' + 4y = 0,$$
$$y(0) = 1,$$
$$y'(0) = 0.$$

Is it possible that there is more than one solution to the initial value problem?

Solution: the characteristic polynomial of the differential equation has a double root at 2, so the general solution to the equations is given by $e^{2t}(C_1 + C_2 t)$. Plugging in $y(0) = 1$ and $y'(0) = 0$ gives the system

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

and so $C_1 = 1$ and $C_2 = -2$; so $y(t) = e^t(1 - 2t)$.

As before, (and in fact for any linear homogenous constant coefficient equation), it is not possible that there is more than one solution to the initial value problem.