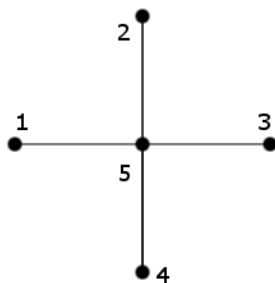


### Problem Set 8, due Friday, June 5

1. Consider the random walk on an  $n$  point circle, except now, the probability that the walker makes a jump clockwise in a time interval  $[t, t+h)$  is  $2h$ , while the probability that the walker makes a jump counterclockwise in a time interval  $[t, t+h)$  is  $h$ , and the probability of staying put in the time interval  $[t, t+h)$  is  $1 - 3h$ .
  - i: Find a differential equation for the probability that the wanderer is at node  $j$  at time  $t$ . You do **not** need to prove your equation is correct.
  - ii: As  $t \mapsto \infty$ , what is the probability that the wanderer is at node  $j$ ? Hint: you will need to look at the real parts of the (complex) eigenvalues of the matrix from your differential equation. Find them by guessing the eigenvectors.
2. Consider the 5 node network in the figure. Assume that in a time interval  $[t, t+h)$ , if the wanderer is at the central node, the probability of transfer to each of the outlying nodes is  $h/4$ , and so the probability of staying put is  $1 - h$ ; if at an outlying node, the probability of a move to the center is  $h$ , and so the probability of staying put is  $1 - h$ .



- i: Find a differential equation for the probability that the wanderer is at node  $j$  at time  $t$ . You do **not** need to prove your equation is correct.
- ii: As  $t \mapsto \infty$ , what is the probability that the wanderer is at node  $j$ ?

3. Consider the standard  $n$  point random walk  $p' = Ap$ , where as usual,

$$A = \begin{pmatrix} -2 & 1 & 0 & \cdots & 0 & 1 \\ 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & & \vdots \\ \vdots & & \ddots & & & 0 \\ 0 & & & & & 1 \\ 1 & 0 & \cdots & 0 & 1 & -2 \end{pmatrix}.$$

- i: Find a nonconstant initial condition  $x$  so that if  $p(0) = x$ , the point on the discrete circle which is the most likely location of the walker does not change in time; i.e. so that the location of maximum value of  $p$  is constant in time.
- ii: Find an initial condition  $z$  so that if  $p(0) = z$ , the point on the discrete circle which is the most likely location of the walker does change in time. You may assume the system has a large number of points if it makes it easier for you; note that it is necessary that the number of points be greater than 3.

For both parts of the question, justify your answers.