Problem Set 5, due Friday, May 15

From section 9.2: 8, 24, 36.

Some Romeo and Juliet problems, taken from Nonlinear Dynamics and Chaos by S. Strogatz... Suppose that $R(t)$ is Romeo's affection for Juliet; and $J(t)$ is Juliet's affection for Romeo. A simple model for their relationship is given by

$$\begin{array}{l} \frac{\partial R}{\partial t} = bJ \\ \frac{\partial J}{\partial t} = cR \end{array},$$

where b and c are constants. Thus the change in Romeo's affection for Juliet depends on her affection for him, and vice versa. If $b > 0$, and $c < 0$, what happens, independent of initial conditions? How do the initial conditions affect the outcome of the affair? What mechanical system is this?

A slightly more complicated model is

$$\begin{array}{l} \frac{\partial R}{\partial t} = aR + bJ \\ \frac{\partial J}{\partial t} = cR + dJ \end{array}.$$

If

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

and Romeo and Juliet start off liking each other a little: $R(0) = J(0) = 1$, what happens? What if $R(0) = -1$, and $J(0) = 1$? If

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix},$$

and $R(0) = J(0) = 1$, what happens? What if $R(0) = -1$, and $J(0) = 1$? Is there some initial condition under which the affair degenerates into complete apathy?

For each of the systems given above, write an explicit general solution. It may help your intuition to play with pplane, the toy linked to on the class webpage.