Problem Set 9, due Friday, March 13

1: Find a minimizer for

$$||x||^2$$

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ -x_2 &\geq -6 \end{aligned}$$

and show that the first and second order sufficient conditions hold at the minimizer. Even if your geometric intuition makes it unnecessary, make sure you check all possible combinations of active constraints for the minimizer.

2: Let Q be a symmetric $n \times n$ matrix, and A a full rank $m \times n$ matrix with $m < n$. Show that a local minimizer for the equality constrained problem

$$x^T Q x$$

$$Ax = b$$

is a global minimizer. Is this true for the problem

$$x^T Q x$$

$$Ax \geq b?$$

Either show it is true or find a counterexample.

3: Suppose f is a smooth function on $\mathbb{R}^n$, A an $m \times n$ matrix, b an m vector, and x_* is a local minimum for

$$f(x)$$

$$Ax \geq b.$$

Show that the Lagrange multipliers corresponding to the active rows of the equation $Ax_* \geq b$ are greater than or equal to 0. what if $\geq$ in the constraints is replaced by $\leq$? What if minimum is replaced by maximum?