Problem Set 8, due Friday, March 6

1: Let A be an $m \times n$ matrix with linearly independent rows, and $n > m$. Find the nearest point to the origin on the affine subset of $\mathbb{R}^n$ given by $x : Ax = b$.

2: Let $f : \mathbb{R}^n \mapsto \mathbb{R}$, $\overline{x} \in \mathbb{R}^n$, and Z an $n \times r$ matrix; let $\phi(v) = f(\overline{x} + Zv)$. Calculate $\nabla\phi$ and $\nabla^2\phi$.

3: Let $x \in \mathbb{R}^n$, and Z an $n \times r$ basis for the null space of the $m \times n$ matrix A. Show that $Z^T x = 0$ is equivalent to the existence of an m vector λ such that $x = A^T\lambda$.

4: Find

$$\max\ x_1 x_2 ... x_n$$

subject to

$$x_1/a_1 + x_2/a_2 + ... + x_n/a_n = 1,$$

where $a > 0$. Make sure you carefully check that you have found a maximum.