Problem Set 7, due Friday, May 23

1: Let Q be a symmetric $n \times n$ matrix, and let c be an n (column) vector. Find the gradient and Hessian of the function $w(x) = x^T Q x + c^T x$. Find necessary and sufficient conditions for w to have a minimum at some point x. Your conditions should depend on Q, x and c...

2: Use the chain rule (even if you think it is easier a different way) to prove the product rule: if f and g are smooth functions on $\mathbb{R}^n$,

$$\nabla fg = g\nabla f + f\nabla g.$$

3: Write a matlab program that takes as input a 4th degree polynomial in two variables (from $\mathbb{R}^2$ to $\mathbb{R}$) and finds a local minimum using Newton's method. Your code from last week will be helpful... Decide on a convergence criterion, and either output the location of the minimum if the method converges, or a non-convergence flag.

4: Write matlab code to find the best 3d order polynomial fit to 1d data X and Y in the sense of least squares. That is: find a_0, a_1, a_2, a_3 minimizing

$$\sum_i (a_0 + a_1 x_i + a_2 x_i^2 + a_3 x_i^3 - y_i)^2.$$

Your code should not use matlab's built in least squares solvers!