We develop a correspondence between the study of Borel equivalence relations induced by closed subgroups of $S_\infty$, and the study of symmetric models of set theory without choice, and apply it to solve questions of [HKL98].

In [HKL98], the possible values of potential complexity of Borel equivalence relations which are induced by actions of closed subgroups of $S_\infty$ are completely classified. To that end, they refine the Friedman-Stanley hierarchy $\in\alpha$, $\alpha<\omega_1$, by defining equivalence relations $\in\lambda+1$, $2\leq\lambda<\omega_1$. E.g., $\in\alpha$ has potential complexity $D(\Pi^0_\alpha)$.

Moreover, they define equivalence relations $\in\lambda+1$, $0\leq\lambda<\omega_1$, and show that they correspond to actions by “well-behaved” closed subgroups of $S_\infty$. That is, if $E\leq B\in\lambda+1$ is induced by a Borel $G$-action of a closed subgroup $G$ of $S_\infty$ which admits an invariant compatible metric, then $E\leq B\in\lambda+1$. Furthermore, they prove that for any countable ordinal $\alpha$, $\in\alpha+3<\omega\in\alpha+3$.

They ask whether the remaining reductions are also strict, and conjecture that they are. We confirm this, and focus on the minimal open cases:

**Theorem 1.** $\in\omega+1< B\in\omega+1$ and $\in\omega+2< B\in\omega+2$.

The proof goes through studying symmetric models generated by generic invariants of these equivalence relations. To make the connection with Borel reducibility we use tools developed by Zapletal and Kanovei-Sabok-Zapletal.

We use models from [Mon73] separating the generalized Kinna-Wagner principles, KWP$^n$, which state that every set can be embedded into the $n$th-power set of an ordinal. Towards proving Theorem 1, we extend Monro’s construction past $\omega$, answering a question of [Kar18].

