▸ ASSAF SHANI, *Borel equivalence relations and symmetric models.*
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We develop a correspondence between the study of Borel equivalence relations induced by closed subgroups of S_∞, and the study of symmetric models of set theory without choice, and apply it to solve questions of [HKL98].

In [HKL98], the possible values of *potential complexity* of Borel equivalence relations which are induced by actions of closed subgroups of S_∞ are completely classified. To that end, they refine the Friedman-Stanley hierarchy $\cong_\alpha$, $\alpha < \omega_1$, by defining equivalence relations $\cong^*_{\lambda+1}$, $2 \leq \lambda < \omega_1$. E.g., $\cong^*_n$ has potential complexity $D(\Pi^0_n)$.

Moreover, they define equivalence relations $\cong^*_{\lambda+1,0} \leq_B \cong^*_{\lambda+1}$, and show that they correspond to actions by "well-behaved" closed subgroups of S_∞. That is, if $E \leq_B \cong^*_{\lambda+1}$ is induced by a Borel G-action of a closed subgroup G of S_∞ *which admits an invariant compatible metric*, then $E \leq_B \cong^*_{\lambda+1,0}$. Furthermore, they prove that for any countable ordinal α, $\cong^*_{\alpha+3,0} <_B \cong^*_{\alpha+3}$.

They ask whether the remaining reductions are also strict, and conjecture that they are. We confirm this, and focus on the minimal open cases:

THEOREM 1. $\cong^*_{\omega+1,0} <_B \cong^*_{\omega+1}$ *and* $\cong^*_{\omega+2,0} <_B \cong^*_{\omega+2}$.

The proof goes through studying symmetric models generated by generic invariants of these equivalence relations. To make the connection with Borel reducibility we use tools developed by Zapletal.

We use models constructed in [Mon73], separating the "*generalized Kinna-Wagner principles*", KWP^n, which state that every set can be injected into the n'th-power set of an ordinal. Towards proving Theorem 1, we show the consistency of $\mathrm{KWP}^{\omega+1} \wedge \neg\mathrm{KWP}^\omega$, answering a question of [Kar1?].

[HKL98]G. HJORTH, A. S. KECHRIS AND A. LOUVEAU, *Borel equivalence relations induced by actions of the symmetric group*, ***Ann. Pure Appl. Logic***, 92 (1998), 63-112.

[Kar1?]A. KARAGILA, *Iterating Symmetric Extensions*, ***ArXiv 1606.06718***.

[Mon73]G. P. MONRO, *Models of ZF with the same sets of sets of ordinals*, ***Fund. Math.***, 80 (1973), no. 2, 105-110.