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TERMINAL NOTIONS

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Abstract. Certain set theoretical notions cannot be split into finer subnotions.

§1. Introduction.

In this paper I wish to comment on a pattern of behavior common to many mathematical notions in use today. Namely, it appears that certain notions cannot be naturally subdivided to more specific notions, or restated, all representatives of such notions share the same natural properties. Here by a notion I mean any formula of one free variable, as in $\phi(x) =$ "x is a Lie algebra", $\phi(x) =$ "x is an ultrafilter on natural numbers".

VAGUE DEFINITION 1. A notion $\phi(x)$ is *terminal* if for every natural property $\psi(x)$ we have

$$\forall x, y\ (\phi(x) \wedge \phi(y)) \to (\psi(x) \leftrightarrow \psi(y)).$$

For example the notion of a group ($\phi(x) =$ "x is a group") is not terminal since there are many different kinds of groups: finite, infinite, torsion, torsionless etc. But what is an example of a terminal notion and what is the place of such notions in mathematics?

Imagine your friend group theorist trying to verify a certain property $\psi(x)$ for groups. Perhaps it holds for all of them, perhaps for some or none at all and he wants to know. In such a case, he will frequently try to find a particular class of groups for which ψ will be easy to verify, by going into more and more specific subcases. "Rule out finite groups, for them ψ is irrelevant. Rule out nonabelian groups, they are too hard to deal with ...." One possible outcome of this procedure is hitting a class C of groups for which your friend holds the following belief:

(B) There are many groups in the class C however I do not know how to subdivide this class in a natural way. ψ will certainly hold the same for all groups in C.

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At this point your friend is well motivated to finally prove something. The class C is a good candidate for a terminal class. Since the above-described procedure is fairly common in mathematical practice, it should be the case that terminal classes proliferate throughout mathematics. However, mathematicians usually have no means to express the fact that a certain notion is terminal, let alone the means of proving it. Thus a belief like (B) is usually viewed merely as a useful superstition. Luckily, it turns out that forcing machinery in conjunction with large cardinals make it possible to isolate many terminal notions and prove precise theorems about them. The purpose of this paper is to illustrate this process on several examples.

The notation in this paper is fairly standard. A passive knowledge of the nonstationary tower techniques as described in [6] is required for understanding Section 4. By a property I always mean a formula of set theory with one free variable, with possible real parameters.

§2. An example. Consider Ramsey ultrafilters. An ultrafilter U on natural numbers is *Ramsey* if for every partition of pairs of natural numbers into two classes there is a homogeneous set in U. Every topologist "knows" that the notion of a Ramsey ultrafilter is terminal in the sense of Vague Definition 1 if we consider only properties ψ invariant under the Rudin-Keisler equivalence $\equiv$ of ultrafilters. Here $U \equiv V$ if there is a permutation π of natural numbers such that $U = \{\pi''a : a \in V\}$. While the existence of Ramsey ultrafilters is unprovable in ZFC—one needs to invoke the Continuum Hypothesis or similar principles to produce them—they still serve as a good source of examples and as a reliable methodological tool. The following three theorems quantify the belief (B) for Ramsey ultrafilters precisely.

THEOREM 2. *Suppose large cardinals exist and $\psi(x)$ is a projective property of ultrafilters that respects the Rudin-Keisler equivalence. Then*

- *either ψ holds in all set generic extensions of all their Ramsey ultrafilters*
- *or $\neg\psi$ holds in all set generic extensions of all their Ramsey ultrafilters.*

Here a property ψ is *projective* if its quantifiers range over real numbers only, and it respects the RK equivalence (or is invariant under it) if ZFC proves "for ultrafilters x, y on natural numbers, $x \equiv y \rightarrow (\psi(x) \leftrightarrow \psi(y))$". For example the property $\psi(x) =$ "x is a Ramsey ultrafilter" can be rewritten as a projective property, and it certainly is RK invariant. The most interesting special case of the Theorem is that of the trivial forcing extension. It says that in the presence of large cardinals all Ramsey ultrafilters share the same RK invariant projective properties—even though they do not have to be all Rudin-Keisler equivalent. The Theorem was known previously to Todorcevic [5]. The large cardinal assumption needed is roughly that of a class of Woodin cardinals. Some such assumption is certainly needed since for example in the constructible universe there is a Δ^1_2 Ramsey ultrafilter and consequently a projectively definable Rudin-Keisler equivalence class of

Ramsey ultrafilters. Laflamme [10, 2] constructed a model where all Ramsey ultrafilters share the same projective topological properties, starting from a Mahlo cardinal.

THEOREM 3. *Suppose large cardinals exist and $\psi(x)$ is a Σ^2_1 property of ultrafilters that respects the Rudin-Keisler equivalence. Then*

- *either ψ holds in all set generic extensions satisfying the Continuum Hypothesis of all their Ramsey ultrafilters*
- *or $\neg\psi$ holds in all set generic extensions of all their Ramsey ultrafilters.*

Here Σ^2_1 properties are those of the form $\psi(x) = \exists Y \subset \mathbb{R}\ \theta(x, Y)$ where θ is projective. Again, the trivial forcing extension case of the Theorem says that in the presence of large cardinals and the Continuum Hypothesis all Ramsey ultrafilters share the same topological Σ^2_1 properties. Thus by strengthening the hypotheses on the universe with which we are working, the class of properties shared by all Ramsey ultrafilters has increased. A good example of a Σ^2_1 topological property is $\psi(x) =$ "x is not a $P_{\aleph_1}$-point" [3]. It was proved by Louveau [11] that under Martin's Axiom and the negation of the Continuum Hypothesis there are both Ramsey $P_{\aleph_1}$-points and Ramsey non-$P_{\aleph_1}$-points and so in the absence of CH Ramsey ultrafilters can disagree on Σ^2_1 properties. One can use the methods of [1] to produce a model of the Continuum Hypothesis where Ramsey ultrafilters do not share Δ^2_2 properties, and so the Theorem cannot be strengthened. The large cardinal assumption needed for the theorem is roughly a class of measurable Woodin cardinals.

THEOREM 4. *Suppose large cardinals exist. Then there is a set generic extension called the homogeneous model satisfying $\Diamond$ in which for every property $\psi(x)$ of ultrafilters respecting the Rudin-Keisler equivalence—without any syntactical restrictions—*

- *either ψ holds of all Ramsey ultrafilters*
- *or $\neg\psi$ holds of all Ramsey ultrafilters.*

Thus if one is content with a mere consistency result—as opposed to the case of previous theorems where certain natural theories outright implied sharing of all topological properties in certain wide syntactical classes by Ramsey ultrafilters—then it is possible to drop all syntactical restrictions on the properties considered. There are many models satisfying the conclusions of the Theorem, constructed from various large cardinal assumptions or from $AD\mathbb{R} + \theta$ regular. For the case of Ramsey ultrafilters it is enough to start from a weakly compact cardinal, however other terminal notions considered below may require stronger initial assumptions. It is unclear whether any of the scale of models thus obtained has a claim to canonicity. For the purposes of this paper the homogeneous model refers to the one obtained from a supercompact cardinal in [16], and it works for all notions considered below.

§3. **Other examples.** Of course were it the case that the likes of the previous theorems can be proved only for Ramsey ultrafilters the matter would not deserve much attention. However, it turns out that the proofs are flexible enough to be applied to many other notions. Consider the case of Lusin sets. A set $X \subset \mathbb{R}$ is *Lusin* if it is uncountable and has countable intersection with every meager set. Again, the existence of such sets is unprovable in ZFC but under assumptions like CH there are many of them. Lusin sets are a good candidate for a terminal class: the everyday topological practice tells us that they all have roughly the same properties. However, a closer study reveals some possible distinctions:

(1) a Lusin set can concentrate in some small open interval of the real line. This is in some sense irrelevant for topologists, and in this paper by a Lusin set I will always mean one with uncountable intersection with every open set.

(2) there can be strong Lusin sets [13] and the ones that are not strong. Here a set $X \subset \mathbb{R}$ is *strong Lusin* if for every meager finitary relation $M \subset \mathbb{R}^n$ there is a countable set $Y \subset X$ such that no n-tuple of distinct elements of the set $X \setminus Y$ belongs to the relation M. It is not known whether the existence of Lusin sets implies the existence of strong Lusin sets.

(3) there can be extendible strong Lusin sets and possibly nonextendible ones. Here extendibility is a rather complicated projective property of the set, but it is implied by and in the presence of CH it is equivalent to the existence of an inclusion-larger strong Lusin set such that the set difference of the two is uncountable. At this point I do not know whether there can be nonextendible strong Lusin sets, so possibly the strengthening of strong Lusin sets to the extendible strong Lusin sets is trivial.

Of course there are other possible distinctions between Lusin sets but they are mostly inconsequential to a topologist: Theorems 2, 3 and 4 hold with Ramsey ultrafilters replaced by extendible strong Lusin sets and with the Rudin-Keisler equivalence replaced by $X \equiv Y$ if the symmetric difference of the two is countable. In fact it is possible to use other equivalences as well, for example $X \equiv Y$ if there is a homeomorphism of the reals mapping X onto Y.

Another typical situation arises with Souslin trees. There are many different kinds of Souslin trees, so one needs to be more specific if a terminal notion is supposed to result. Consider the free trees [1, 4, 12]: an ω_1-tree is *free* if for every finite tuple $t_0 \dots t_n$ of incomparable elements of T the poset $T \restriction t_0 \times \dots \times T \restriction t_n$ has no uncountable antichain. While free trees are Souslin, not every Souslin tree is free and it is unknown whether the existence of Souslin trees implies the existence of free trees. Theorems 2, 3, and 4 hold with Ramsey ultrafilters replaced by free trees and the Rudin-Keisler equivalence replaced by $T \equiv S$ if the complete boolean algebras determined by the two trees are isomorphic.

The last example of a terminal notion in this paper are diamond sequences. Again, there are many different types according to which objects of size $\aleph_1$ they are supposed to guess, so it is necessary to invent sequences which are in some sense universal. A sequence $\langle r_\alpha : \alpha \in \omega_1 \rangle$ of real numbers is a *good diamond sequence* if for every borel relation $B \subset \mathbb{R}^{\aleph_0} \times \mathbb{R}$, every sequence $\langle s_\alpha : \alpha \in \omega_1 \rangle$ of reals and every club $C \subset \omega_1$ there is an ordinal $\alpha \in C$ such that either there is no real r such that $\{ s_\beta : \beta \in \alpha \} B\, r$ or else r_α is a real such that $\{ s_\beta : \beta \in \alpha \}\ B\ r_\alpha$. While this definition may sound confusing, it is a straightforward matter to show that $\diamondsuit$ is equivalent to the existence of good diamond sequences. Theorems 2, 3, and 4 hold with Ramsey ultrafilters replaced by good diamond sequences and the Rudin-Keisler equivalence replaced with the equality on a closed unbounded set.

Some of the above homogeneity results can be combined in a quite entertaining way: one can show that in the homogeneous model, if $\psi(U, T, S)$ is a formula about a Ramsey ultrafilter, Souslin tree and a stationary subset of ω_1 which respects the Rudin-Keisler equivalence of filters, the boolean algebra equivalence of trees and modulo nonstationary ideal equality of sets, then the formula holds the same of all combinations of a Ramsey ultrafilter, free tree and a stationary costationary subset of ω_1. One way to state this result is "Ramsey ultrafilters, free trees and stationary sets have nothing to do with each other".

§4. The proofs.

The proofs of the absoluteness theorems vary slightly from case to case. The idea can be best explained on the case of Ramsey ultrafilters. The most important observation here is that it is possible to adjoin a Ramsey ultrafilter to the universe as a generic filter for the poset $\mathcal{P}(\omega)/\mathrm{fin}$. Now suppose U is a Ramsey ultrafilter and δ is a suitable large cardinal. The necessary large cardinal strength of δ can be precisely calibrated, for the sake of brevity I assume below that δ is supercompact. The key claim:

CLAIM 5. *The nonstationary tower forcing $\mathbb{Q}_{<\delta}$ forces the following*:

(1) $\dot{\mathbb{R}}$ *are the reals of some external Levy collapse of δ to ω_1*
(2) $V(\dot{\mathbb{R}})[j\check{U}]$ *is a $\mathcal{P}(\omega)/\mathrm{fin}$ extension of $V(\dot{\mathbb{R}})$*

where $\dot{\mathbb{R}}$ are the reals of the extension and $j\colon V \to M$ is the generic ultrapower.

It is immediately clear from the claim that for a projective property $\psi(x)$ of ultrafilters respecting the Rudin-Keisler equivalence,

$$\psi(U) \leftrightarrow \mathrm{Coll}(<\delta) * \mathcal{P}(\omega)/\mathrm{fin} \Vdash \psi(\dot{U}_{\mathrm{gen}}) \tag{4}$$

where $\dot{U}_{\text{gen}}$ is the name for the generic Ramsey ultrafilter added by the second step of the iteration. The equivalence (4) holds since

$$\begin{aligned} V \models \psi(U) &\leftrightarrow M \models \psi(jU) \\ &\leftrightarrow V(\mathbb{R})[jU] \models \psi(jU) \\ &\leftrightarrow \text{Coll}(<\delta) * \mathcal{P}(\omega)/\text{fin} \Vdash \psi(\dot{U}_{\text{gen}}). \end{aligned}$$

Here the second and third equivalences follows from the fact that the model M has the same reals as the $\mathbb{Q}_{<\delta}$-extension, and so must agree with the model $V(\mathbb{R})[jU]$ on projective properties such as ψ. The last equivalence is the consequence of the forcing theorem and the fact that for Rudin-Keisler invariant properties all conditions in the forcing decide their validity for $\dot{U}_{\text{gen}}$ in the same way.

Theorem 2 now follows from the fact that the validity of the formula on the right of (4) does not depend on the particular choice of the ultrafilter U or the forcing extension as long as it was obtained via a poset of size $< \delta$.

For Theorem 3 one needs to compute the remainder forcing $\mathbb{Q}_{<\delta}/\dot{\mathbb{R}}, jU$ and show that in the presence of a suitable V-generic Radin sequence converging to δ this forcing has a σ-closed subset. It then turns out that for any Σ^2_1 property ψ respecting the Rudin-Keisler equivalence,

$$\text{(5)} \qquad \text{(CH)} \quad \psi(U) \leftrightarrow R * \text{Coll}(<\delta) * \mathcal{P}(\omega)/\text{fin} \Vdash \psi(\dot{U}_{\text{gen}})$$

where R is a Radin forcing with a sequence of measures at δ with a repeat point. For the left-to-right direction one proves that in a $R * \text{Coll}(<\delta) * \mathcal{P}(\omega)/\text{fin}$ extension $V[H]$ there is a V-generic filter $G \subset \mathbb{Q}_{<\delta}$ such that $\mathbb{R} \cap V[G] = \mathbb{R} \cap V[H]$ and $j(U) = U_{\text{gen}}$ where $j : V \to M$ is the generic ultrapower. Then

$$\begin{aligned} V \models \psi(U) &\to M \models \psi(j(U)) \\ &\to V[H] \models \psi(j(U)) \\ &\to R * \text{Coll}(<\delta) * \mathcal{P}(\omega)/\text{fin} \Vdash \psi(\dot{U}_{\text{gen}}). \end{aligned}$$

The first implication uses the elementarity of j and the second uses the syntactical form of ψ:

$$\psi(U) = \exists X \subset \mathbb{R}\ \theta(X, U)$$

for some projective formula θ and so the witness $X \subset \mathbb{R}$ for $\phi(jU)$ in the model M will be such a witness in the model $V[H]$.

For the right-to-left direction one applies CH, the technology of Woodin [14] and Claim 5 to find a generic embedding $i : V \to N$ and a V-generic filter

$$H \subset R * \text{Coll}(<\delta) * \mathcal{P}(\omega)/\text{fin}$$

in the model N such that $\mathbb{R} \cap V[H] = \mathbb{R} \cap N$ and $i(U) = U_{\text{gen}}$. Then

$$V[H] \models \psi(U_{\text{gen}}) \to N \models \psi(i(U)) \to V \models \psi(U)$$

where the first implication follows from the syntactical form of ψ as before. The sandwich diagram below sums up the idea from this argument:

$$\begin{array}{ccccc}
V & & & & V \\
{\scriptstyle j}\big\downarrow & & & & {\scriptstyle i}\big\downarrow \\
M & \xrightarrow{\subseteq} & V[H] & \xrightarrow{\subseteq} & N \\
\mathbb{R}\cap M & = & \mathbb{R}\cap V[H] & = & \mathbb{R}\cap N \\
j(U) & = & U_{\text{gen}} & = & i(U)
\end{array}$$

Now Theorem 3 obtains: the expression on the right hand side of (5) does not depend on the particular choice of the ultrafilter U and the set generic extension we choose to work in as long as it was obtained via a forcing of size $< \delta$. Thus all set generic extensions of CH agree on the Σ^2_1 topological properties of their Ramsey ultrafilters. The proof is completed by noting that if a Σ^2_1 property holds of a Ramsey ultrafilter in some extension then it holds in an extension satisfying the Continuum Hypothesis—the one obtained via a σ-closed collapse of the continuum to $\aleph_1$.

The absoluteness theorems for the other notions are demonstrated using the same scheme, only a different σ-closed forcing is used to obtain a generic instance of the notions in question. The key point always is the possibility to prove a version of Claim 5.

The notationally simplest way to obtain a homogeneous model of Theorem 4 is to start from a supercompact cardinal κ and a larger inaccessible cardinal λ. In such a situation one can find normal measures U_α on $\mathcal{P}_{<\kappa}\alpha$ for ordinals $\alpha \in \lambda$ which project onto each other, and construct the model

$$V(\mathbb{R})[\, C_\alpha : \alpha \in \lambda\,]$$

where $\mathbb{R}$ are the reals of a Levy collapse of κ to ω_1 and $\langle\, C_\alpha : \alpha \in \lambda \rangle$ is a sequence of generic inclusion-increasing continuous towers of sets diagonalizing the measures U_α. That is, the sets $\langle\, C_\alpha : \alpha \in \lambda\rangle$ are generic for the poset

$$P = \prod_{\alpha\in\lambda} P_\alpha,$$

where

$$\begin{aligned} P_\alpha = \{\, \langle f, X\rangle : f : \gamma + 1 \to (\mathcal{P}_{<\kappa}\alpha)^V &\text{ is an inclusion-increasing} \\ &\text{and continuous function for some ordinal } \gamma \in \kappa, \text{ and } X \in U_\alpha \,\} \end{aligned}$$

with the ordering $\langle f, X\rangle \geq \langle g, Y\rangle$ if $f \subset g$, $Y \subset X$ and $\operatorname{rng}(g \setminus f) \subset X$. The function f in a condition indicates an initial segment of the generic tower C_α. The product in the definition of the poset P is taken with countable support, and the whole definition of P takes place in $V(\mathbb{R})$.

This is the homogeneous model. It is not hard to see that

$$V(\mathbb{R})[\,C_\alpha : \alpha \in \lambda\,] \models \mathrm{ZFC} + \Diamond + \kappa = \omega_1 + \lambda = \omega_2$$

and for every set $X \subset \mathbb{R}$ in $V(\mathbb{R})[\,C_\alpha : \alpha \in \lambda\,]$ there is an ordinal β such that $X \in V(\mathbb{R})[\,C_\alpha : \alpha \in \beta\,]$. To argue for the homogeneity properties of Ramsey ultrafilters in this model, one follows closely the proof of the Claim 5 to show that whenever U is a Ramsey ultrafilter and $\beta \in \lambda$ is an ordinal such that $U \in V(\mathbb{R})[\,C_\alpha : \alpha \in \beta\,]$ then

$$V(\mathbb{R})[\,C_\alpha : \alpha \in \lambda\,] = V(\mathbb{R})[U][\,C_\alpha : \beta \in \alpha \in \lambda\,][K] \tag{6}$$

where U is a $V(\mathbb{R})$-generic subset of $\mathcal{P}(\omega)/\text{fin}$, $\langle\, C_\alpha : \beta \in \alpha \in \lambda\,\rangle$ is a $V(\mathbb{R})[U]$-generic subset of $\prod_{\beta\in\alpha\in\lambda} P_\alpha$ and K is a $V(\mathbb{R})[U][\,C_\alpha : \beta \in \alpha \in \lambda\,]$-generic subset of $\mathrm{Add}(1, \aleph_1)$. The key point in this decomposition is the fact that K can be obtained as a generic filter for a σ-closed poset. The homogeneity properties of the model now follow from the homogeneity of the forcings involved in the right hand side of (6).

The proof of homogeneity properties of the model for other terminal notions uses the same idea peppered with a trick or two specific to the notion to obtain a decomposition like the one in (6). In the case of good diamond sequences one appears to need the assumption that the measures U_α come from an almost huge embedding j with critical point κ and $j(\kappa) = \lambda$.

§5. **Open problems.** There is a very attractive though unlikely possibility that every notion concerning sets of reals has an interesting terminal strengthening. This is difficult to formulate precisely since one has to choose suitable equivalences together with the notions. I do not know of a good terminal strengthening to $\phi(x) =$ "x is a special Aronszajn tree".

Another interesting problem concerns filling in the following table:

LC	**LC + CH**	**LC + CH + $\Diamond$ + ?**	...	**a specific model**
projective ψ	$\Sigma^2_1\ \psi$	$H_{\aleph_2} \models \psi$	...	no restriction

where the top row entries are theories needed to obtain homogeneity for terminal notions with respect to properties in the syntactical class indicated in the bottom row. The third column would be of particular interest since under the Continuum Hypothesis most real line topology takes place in the model $H_{\aleph_2}$. Given the universal inclination among topologists to resort to a model of $\Diamond$ if things are supposed to go particularly smoothly one would expect the theory in question to be a slight strengthening of $\Diamond$.

Lastly, one can observe terminal phenomena in a somewhat different context. For example consider the common phrase "choose a fast club subset of ω_1". Can it perhaps be the case that the theory of $\langle H_\kappa, \in, C\rangle$ stabilizes on a cone of clubs C, i.e., is there a club $D \subset \omega_1$ such that for clubs $C_0, C_1 \subset D$ we always have $\langle H_\kappa, \in, C_0\rangle \equiv \langle H_\kappa, \in, C_1\rangle$?

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