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The Annual Logic Conference was held at the University of Auckland as part of Philfest-97. Four organizations, the Australasian Association for Logic, the Australasian Association for Philosophy, Women in Philosophy, and the Australasian Association for the History, Philosophy, and Social Studies of Sciences, all cooperated to confer in Auckland during July 5-13, 1997. Over 250 attended Philfest-97. There were 32 registrations for the AAL Conference. The Secretary of both Philfest-97 and the Logic Conference was Rod Girle.

The Presidential Address, *An experimentation in formalised metalogic: From an investigation into the relationship between closure operators, closure systems, consistency, Lindenbaum operators and maximal families of sets*, was given by Stan Surma.

The Annual General Meeting of the AAL agreed that the next Conference should be at Macquarie University in Sydney, and that it should be a Richard Sylvan Memorial Conference.

Abstracts of contributed talks given (in person or by title) follow.

For the ASL Committee on Logic in Australasia
ROD GIRLE

Abstracts of contributed talks

► NICOLETTE BONNETTE AND RAJEEV GOR'E, *Labelled sequent systems for tense logics*.
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We extend the well known labelled tableau method for modal logics to tense logics, thereby giving the proof-theoretical foundations for a modular theorem prover for tense logics. Labels allow us to capture the Kripke semantics of tense logics directly in our syntax. Specifically, we extend Fitting's labelled sequents for modal logics to handle the converse of accessibility, thereby obtaining a labelled sequent system for the basic tense logic $\mathbf{K_t}$. We then show how to extend this basic system by adding inference rules corresponding to various basic restrictions on accessibility like seriality, reflexivity, transitivity, symmetry, and euclideaness. The given rules can be combined in a modular fashion to give labelled sequent systems for numerous decidable extensions of $\mathbf{K_t}$. We are currently studying the computational properties of our systems in order to turn them into decision procedures. The structure of our sequents, and the form of the rules, makes it easy to implement them in Prolog to give efficient therem

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provers for many tense logics.

► C. S. CALUDE, E. CALUDE, B. KHOUSSAINOV, AND M. LIPPONEN, *From complete to incomplete automata: Simulation, universality and minimality*.
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Finite automata have been used as alternative, discrete models in theoretical physics [5, 1, 2]. The automaton is assumed to be a "black box", with only the input and output terminals accessible to the experimenter. In [4, 3, 1] one proves that *it is impossible to determine the initial state of an automaton*, hence we study finite deterministic automata *without initial states*. The response function of an automaton is the sequence of outputs produced by the states visited during a computation. Two automata are *equivalent* if they produce the same outputs. An automaton $\mathcal{A}$ is *universal* for a finite class C of pairs of automata and initial states if $\mathcal{A}$ produces the same outputs as any element of C and nothing more. Main results: (a) *Two automata are equivalent iff they are universal for the same class*. (b) *Two equivalent minimal automata are isomorphic*. If some internal transitions are *undefined*, then we are led to the notion of *incomplete* automaton without initial states. *All results above stated for complete automata hold true also for incomplete automata*. Further on, we say that an automaton $\mathcal{A}$ satisfies CI if it is minimal and there exists a state such that no single experiment can distinguish it from all the other states; $\mathcal{A}$ satisfies CII if for every state there is an experiment distinguishing it from all the other states, but there is no unique experiment distinguishing any two different states, [1]. Properties CI/CII cannot hold for a complete automaton with less than four states; however, there exist three-state incomplete automata working on a singleton alphabet and having CI/CII.

[1] Cristian Calude, Elena Calude, Karl Svozil and Sheng Yu, *Physical versus computational complementarity* I, ***International Journal of Theoretical Physics***, vol. 36 (1997), pp. 1495–1523.

[2] Elena Calude and Marjo Lipponen, *Minimal deterministic incomplete automata*, ***Journal of Universal Computer Science***, vol. 3 (1997), pp. 1194–1198.

[3] Gregory J. Chaitin, *An improvement on a thereom by E. F. Moore*, ***IEEE Transactions on Electronic Computers***, vol. EC-14 (1965), pp. 466–467.

[4] Edward F. Moore, *Gedanken-experiments on sequential machines*, In C. E. Shannon and J. McCarthy, editors, ***Automata Studies***, Princeton University Press, Princeton, 1956.

[5] K. Svozil, *Randomness & Undecidability in Physics*, ***World Scientific***, Singapore, 1993.

► R. COLES, R. DOWNEY, AND B. M. KHOUSSAINOV, *On initial segments of computable linear orders*.
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Our basic interest is motivated by the old problem about understanding the isomorphism types of initial segments of computable linearly ordered sets. We recall that a linearly ordered set $(L, \leq)$ is **computable** if the set L and the order relation $\leq$ are computable sets. If the isomorphism type of a linearly ordered set $(L, \leq)$ contains a computable linear oder, then $(L, \leq)$ is **computably presentable**. One of the basic questions concerning computable mathematical structures (in our case linearly ordered sets) is to understand the relationship between classical isomorphism types and computable isomorphism types. We refer the interested reader to Downey [3] for a survey paper on computable linearly ordered sets.

If $(L, \leq)$ is computable linear order then it is natural to ask whether its given initial segment is computable or computably presentable. In general, computable linear orders can possess quite complicated initial segments from a computability point of view. For example, the first constructive ordinal ω_1^{ck} can be an initial segment of a computable linear order. Hence it is possible for $L_1 + L_2$ to be a computable linear order and yet L_1 to be not even hyperarithmetical.

Raw in [2] proved that if $(L, \leq)$ is a $\prod_1^0$–initial segment of a computable linear order with a computable adjacency relation, then $(L, \leq)$ in fact has a computable presentation. On the other hand, Raw [2] constructed an example of a computable linearly ordered set with a $\prod_3^0$–initial segment without computable presentations. Recently, Ambos-Spies, Cooper and Lempp [1] improved the positive direction of Raw's result. They proved that every Σ_2^0–initial segment of a computable linear order possesses a computable presentation. We close the circle by proving the following theorem which uses a $0'$–priority argument construction.

Theorem: There exists a computable linearly ordered set with a $\prod_1^0$–initial segment which does not posess a computable presentation.

[1] K. Ambos-Spies, S. B. Cooper, and S. Lempp, *Initial segments of linear orders*. To appear.

[2] M. J. S. Raw, *Complexity of automorphisms of recursive linear orders* Ph.D. Thesis, University of Wisconsin-Madison (1995).

[3] R. Downey, *Recursion theory and linear orderings*, Handbook of Computable Algebra. Editors Yu. Ershov, S. S. Goncharov, A. Nerode and J. Remmel.

► NEIL LESLIE, *Mutual recursion in Martin-Löf's type theory*.
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In this paper we show how to add mutually recursive types to Martin-Löf's type theory. The type theory of Per Martin-Löf is an extensible theory, in the sense that, when we use the theory we are free to add new types. This is one of the attractions of this theory for the computer scientist who wishes to specify and derive (proven correct) algorithms. We may not add new types completely freely of course, as we must ensure that the types that we add do not introduce absurdities into the theory. We avoid absurdities by constructing the elimination rule using the introduction rules and the computation rules. Our approach is unusual in that we emphasise the role of the computation rules in justifying the inference rules. We show how to add mutually recursive types to the theory directly with the rules justified by our preceding argument. As the rules for these types differ from those proposed in the literature, and possess some unusual features, we show that they are correct by encoding mutually recursive types as general trees.

► GREG RESTALL, *Ways things can't be*.
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In this paper I present an account of impossible worlds which constructs such worlds in a motivated way from entities semanticists and logicians have ready to hand: possible worlds. I show that this conception of impossible worlds provides a semantics for known paraconsistent logics, while avoiding (or perhaps better, while explaining) commitment to strange 'impossibilia' or odd, extra truth values beyond the true and the false. This construction provides an interpretation of paraconsistent semantics in terms that classical logicans can understand and use without rejecting the insights of classical logic.

► J. L SHAW, *The Nyãya on number*.
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The aim of this paper is to discuss (1) whether the Nyãya concept of number is different from logicism, intuitionism and formalism, (2) whether the Nyãya theory can explain certain cognitive diffferences which cannot be explained in terms of set theory, and (3) whether the Nyãya use of 'only' with reference to a number is different from Russell's use of the expression.

A number, in the traditional Nyãya ontology, is a quality of substance, and hence it resides in a substance by the relation of inherence. In order to explain the distinction between the meanings of the word 'two' which has occurred in the sentences 'This object is not two, but it is a locus of two' and 'These are two objects', the Nyãya introduces two types of relation for relating the number two to its loci. Hence there are two uses of the word 'two'. In the former sentence the second occurrence of 'two' refers to a quality which is related to its locus by the relation of inherence, but in the latter sentence 'two' refers to a property which is related to the collection by a relation of *paryãpti*. A *paryãpti* relation is a type of self-linking (*svarūpa*) relation in the Nyãya ontology. Since each number two is a particular one, there would be as many particular twos as there are collections of two objects. As 'two' can be used in two senses, so are the other numbers.

In addition to the above two uses of the word 'two', the Nyãya introduces another use of 'two' in sentences such as 'Two persons are in this building'. Here the word 'two' means 'only two'. The Nyãya explains the meaning of the word 'only' in terms of having an absence or a negative property. As 'two' can be used in the sense of 'only two', so are the other numbers.

► JOHN K. SLANEY, ROBERT K. MEYER, AND GREG RESTALL, *Linear arithmetic desecsed.*
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In classical and intuitionistic arithmetics, any formula implies a true equation, and a false equation implies anything. In weaker logics fewer imiplications hold. In this paper we rehearse known results about the relevant arithmetic $\mathbf{R}\#$, and we show that in linear arithmetic $\mathbf{LL}\#$ by contrast false equations never imply true ones. As a result, linear arithmetic is 'desecsed'. A formula A which entails $0 = 0$ is a 'secondary equation'; one entailed by $\sim(0 = 0)$ is a 'secondary unequation'. A system of formal arithmetic is 'secsed' if every extensional formula is either a secondary equation or a secondary unequation. We are indebted to the program MaGIC for the simple countermodel SZ7, on which $0 = 1$ is not a secondary formula. This is a small but significant success for automated reasoning.

► ATWELL TURQUETTE, *A generalization of Tarski's Möglichkeit*.
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In Jan Lukasiewicz's 1930 paper entitled "Philosophical Remarks on Many-Valued Systems of Propositional Logic," an account is given of Alfred Tarski's discovery in 1921 of a definition expressing "general possibility" while he was attending Lukasiewicz's Warsaw seminars. Tarski uses the first letter of Möglichkeit to denote his concept of "general possibility" and defines it in terms of Lukasiewicz's triadic logical operators C and N as follows:

$$Mp =_{df} CNpp.$$

Lukasiewicz concludes that this triadic definition of "general possibility" leaves little room for improvement.

The present paper extends Tarski's definition of M to the m-valued case, where $m > 3$. Unlike Lukasiewicz's and Tarski's modal-logic motivations, however, the paper is essentially motivated by formal considerations. To be sure, the resulting formal constructions suggest modal interpretations which are structurally different from Tarski's. But more important, perhaps, is the possibility of interpreting the constructions as languages for analysing m-state systems. In particular, they may prove useful in the area of complex network design.

► EDWARD N. ZALTA, *Natural numbers as abstract objects*.
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In 1993, Heck showed that Frege validly derived the Peano/Dedekind axioms for number theory from Hume's principle ($\sharp F = \sharp G$ iff F and G are in one-to-one correspondence) without making any essential appeal to Basic Law V or extensions (other than to establish Hume's Principle in the first place). Burgess and Hazen have shown that second-order logic with Hume's Principle is consistent. But Hume's Principle involves the primitive mathematical notion "the number of Fs" and is subject to 'the Julius Caesar problem', among others.

In this talk, I reconstruct an important subset of the theorems in Frege's *Grundgesetze* without appealing to any mathematical primitives, implicit definitions, or principle of infinity. My theory of abstract objects is supplemented with the assumption that Predecessor is a relation and a modal axiom that logicians implicitly accept. The derivations of the Peano/Dedekind axioms reconstruct Frege's derivations, with the exception of the claim that every number has a successor. The derivation of this latter claim appeals to the modal axiom.

Abstract presented by title

► ROBERT K. MEYER, KATALIN BIMBO, AND J. MICHAEL DUNN, *Dual combinators bite the dust*.
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Dunn and Meyer suggested in [1] that, along with ordinary combinators that operate on their arguments, one might have dual combinators, which (so to speak) are arguments working on their functions. We shall use upper-case for ordinary combinators, and lower case for the corresponding dual combinators. Thus where we have

$$Ix = x$$

the reasonable dual law would be

$$xi = x.$$

The first hint that all was not well with this suggestion came from Mariangiola Dezani, who pointed out that the related ideal of "symmetric lambda calculus" produces inconsistency.

Bimbo struck the next blow, noting that Church-Rosser FAILS for CL with inverses. For, thinking from left to right, we have

$$Ii = i.$$

But the right to left thought produces

$$Ii = I.$$

But does Church-Rosser REALLY fail? Its cash value is non-triviality for **CL**. Maybe, with duals, the system is trivial. Let's try invoking the combinator K and its dual k. For, thinking from left to right, we get

$$(Kx)(yk) = x.$$

But right to left gives us

$$(Kx)(yk) = y.$$

Whence elementary properties of = produce $x = y$ for all x, y. Oops!

We note in the conclusion that relevant logicians, from [2] on, have had it in for CANCELLATIVE combinators like K. And rightly so, since Bimbo has shown also that the argument to triviality REQUIRES cancellation!

[1] J. M. Dunn and R. K. Meyer, *Combinators and structually free logic*, ***Logic Journal of the IGPL***, vol. 5 (1997), pp. 505–537.

[2] A. Church, *The calculi of lambda conversion*, Princeton, 1940.