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1998 SPRING MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Los Angeles, California, March 27–28, 1998

The Spring Meeting of the Association for Symbolic Logic was held in Los Angeles, on March 27–28, 1998, in conjunction with the annual meeting of the Pacific Division of the American Philosophical Association. The Program Committee consisted of C. Anthony Anderson, Penelope Maddy, and Donald A. Martin (Chair). The program included a symposium consisting of two invited talks with invited commentators, two other invited talks, six contributed talks, and three papers presented by title only.

The content of the symposium on Topics in Intensional Logic was as follows:

Edward N. Zalta (Stanford), *Natural cardinals and natural numbers as abstract objects*. Commentator: Tony Roy (California State University, San Bernardino).

Michael Jubien (University of California, Davis), *On propositionality and "abstract" worlds*. Commentator: Christopher Menzel (Texas A&M University).

The other invited talks were:

G. Aldo Antonelli (Michigan State University), *The mathematics of defeasible reasoning: Skepticism vindicated*.

James Tappenden (University of Michigan), *Proof style and understanding in mathematics*.

Abstracts of invited and contributed talks (given in person or by title) follow.

For the Program Committee
DONALD A. MARTIN

Abstracts of invited talks

G. ALDO ANTONELLI, *The mathematics of defeasible reasoning: Skepticism vindicated*.
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The emergence of so-called *non-monotonic logics* through the work of, among others, McCarthy, McDermott & Doyle, Moore, and Reiter, is undoubtedly one of the most significant developments both in logic and in artificial intelligence. Among the several formalisms proposed in the literature, Reiter's *default logic* in particular seems to provide a very flexible general-purpose framework. The central notion of default logic is that of *extension*, roughly analogous to a maximally consistent theory of classical logic. However, by their very nature, extensions for default logic are notoriously recalcitrant to an inductive analysis.

This paper aims to impose a non-trivial algebraic structure on the collection of the extensions of a default theory by introducing a generalization of Reiter's notion of "extension", which is inspired by the Kripke-Woodruff-Herzberger fixpoint approach to the theory of truth. In particular, the present approach differs from the standard one in the way conflicts among defaults are handled: According to the "three-valued" intuition, defaults need not be

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explicitly triggered to pre-empt other defaults. As in Kripke's theory of truth, we get minimal, as well as "optimal" (in the sense of Manna & Shamir) extensions for default theories.

There are three major fall-outs of the present proposal, two technical ones and a conceptual one. The first technical fall-out is that default theories are now always guaranteed to have an extension in the new general sense. The second technical fall-out is that it is now possible to define a relation of *defeasible consequence* satisfying Gabbay's three requirements of Reflexivity, Cut, and (crucially) Cautious Monotonicity. Finally, the third conceptual fall-out is that the present approach vindicates the "skeptical" intuition of Horty, Thomason & Touretzky according to which even when reasoning defeasibly, in the presence of conflicting defeasible rules (and in the absense of further considerations such as specificity) it makes sense to withhold commitment to any conclusion.

MICHAEL JUBIEN, *On propositionality and "abstract" worlds*.
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Using a Cantor-style argument and what I think are reasonable property-theoretic assumptions, I argue that there is no property of being a proposition. Then, exploiting this conclusion, I argue that there are no "abstract" possible worlds, that is, worlds of the "maximal consistent proposition" variety. Although the nonexistence of the property of being a proposition does not itself entail that there are no propositions, I will offer some further considerations in support of that conclusion.

EDWARD N. ZALTA, *Natural cardinals and natural numbers as abstract objects*.
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In 1993, Heck showed that Frege validly derived the Peano/Dedekind axioms for number theory from Hume's Principle ($\#F = \#G$ if and only if F and G are in one-to-one correspondence) without making any essential appeal to Basic Law V or extensions (other than to establish Hume's Principle in the first place). Burgess and Hazen have shown that second-order logic with Hume's Principle is consistent. But Hume's Principle involves the primitive mathematical notion "the number of Fs" and is subject to 'the Julius Caesar problem', among others.

In this talk, I derive an important subset of the theorems in Frege's *Grundgesetze* without appealing to any mathematical primitives, implicit definitions, or principle of infinity. The derivations take place within my theory of abstract objects. This is a system that validates Boolos's claim that "it is plain that Frege uses not one but two instantiation relations." The derivations of the Peano/Dedekind axioms reconstruct Frege's derivations, with the exception of the claim that every number has a successor, which is derived from a modal axiom that logicians implicitly accept.

Abstracts of contributed talks

KATALIN BIMBÓ, *Display logic for systems with explicit combinators*.
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Meyer in [6] suggested to replace the usual structural rules of a Gentzen system with explicity combinators. Dunn and Meyer in [4] developed this approach even further allowing combinators to be formulas in a full sense. The systems in [4] have fusion ($\circ$) and implication ($\rightarrow$) as logical connectives, and the paper shows that the Elimination Theorem holds for these systems. Two extensions involving extensional conjunction and disjunction have been considered in [2], again with ET remaining true. However, in all these cases the systems were right singular.

Moving to systems with multiple right hand sides one of the questions to be answered is what are the right introduction rules for combinators. There are two natural ways to replace the structural rules. In one of these, the combinatory rules are literally the same on both sides. ET is false in these systems, and this does not depend on the particular set of combinators.

The papers offers a modification of Display Logic (DL) (introduced by Belnap in [1]) to accommodate explicit combinators in the right singular systems. Combinators are introduced—beyond the reflexivity axiom—by structural rules as structural constants. For each structural constant there is a rule which turns it into a formula. I illustrate the failure of ET in one of the approaches for the multiple right handed system and give a solution to the problem: combinators are introduced as structural constants, but no rule can turn them into formulas. Time permitting I will consider the possibility of having a version of DL for these systems which is closer to the original system of Belnap.

[1] N. Belnap, *Display logic*, *Journal of Philosophical Logic*, vol. 11 (1982), pp. 375–417.

[2] K. Bimbó and J. M. Dunn, *Two extensions of the structurally free logic LC*, 1998, to appear in the special issue of the *Logic Journal of IGPL* on generalized Gentzen systems.

[3] H. B. Curry and R. Feys, *Combinatory logic*, vol. I, North-Holland, Amsterdam, 1968.

[4] J. M. Dunn and R. K. Meyer, *Combinators and structurally free logic*, *Logic Journal of IGPL*, vol. 5 (1997), pp. 505–537.

[5] R. Goré, *Substructural logics on display*, *Technical report*, Automated Reasoning Project, Australian National University, 1997, draft manuscript.

[6] R. K. Meyer, *A general Gentzen system for implicational calculi*, *Relevance Logic Newsletter*, vol. 1 (1976), pp. 189–201.

PHILIP EHRLICH, *Number systems with simplicity hierarchies: A generalization of Conway's theory of surreal numbers*.
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In his monograph *On numbers and games*, J. H. Conway introduced an ordered field *No* consisting, as he aptly quips, of "all numbers great and small." In addition to its distinguished structure as an ordered field, however, *No* has a rich algebraico-tree-theoretic structure, or *simplicity hierarchy*, that emerges from the recursive clauses in terms of which it is defined. In the present paper, we introduce a novel class of structures whose properties generalize those of *No* and explore some of the remarkable relations that exist between *No* and this more general class of *s-hierarchical ordered structures* as we call them. More specifically, we define a number of types of s-hierarchical ordered structures (group; fields; etc.) as well as a corresponding type of *s-hierarchical mapping*, identify *No* as a *complete* s-hierarchical ordered group (ordered field; etc.), and show that there is one and only one s-hierarchical mapping of an s-hierarchical ordered structure into *No* (or any *complete* s-hierarchical ordered structure, more generally). These mappings are found to be monomorphisms of their respective kinds (ordered groups; ordered fields; etc.) whose images are initial subtrees of *No*. This together with the completeness of *No* enables us to characterize *No*, up to isomorphism, as the unique complete as well as the unique *non-extensible* and the unique *universal*, s-hierarchical ordered group (field, etc.). Following this, we turn to uncovering the spectrum of s-hierarchical ordered structures. Given the nature of *No* alluded to above, this reduces to revealing the spectrum of *s-hierarchical substructures* of *No*, i.e., the subgroups, subfields, etc. of *No* that are initial subtrees of *No*. Included among our findings are the following two results: I. *Every divisible ordered abelian group is isomorphic to an s-hierarchical subgroup of No*; II. *Every real-closed ordered field is isomorphic to an s-hierarchical subfield of No*.

DON FALLIS, *The reliability of randomized algorithms*.
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Recently, certain philosophers of mathematics have argued that there are no *epistemic* considerations that should stop mathematicians from using probabilistic methods (such as randomized algorithms) to establish mathematical truths (see, e.g., [1, 2]). However, mathematicians clearly should not use a particular method to establish that a mathematical claim is true if they do not have good evidence that the method is reliable. Unfortunately, as the computer scientist Eric Bach points out, "when a randomized algorithm is implemented, one always uses a sequence whose later values come from earlier ones in a *deterministic* fashion. This invalidates the assumption of independence and might cause one to regard results about probabilistic algorithms with suspicion" ([3], p. 30). In other words, it is not immediately clear that randomized algorithms are reliable *in practice*. In this paper, I argue that there is good evidence for the reliability of randomized algorithms.

[1] DON FALLIS, *The epistemic status of probabilistic proof*, *Journal of Philosophy*, vol. 94, no. 4 (1997), pp. 165–186.

[2] CATHERINE WOMACK and M. FARACH, *Randomization, rigor and persuasiveness in proofs*, delivered at the International Meeting on Logic and Mathematical Reasoning, Mexico City, 1997.

[3] ERIC BACH, *Realistic analysis of some randomized algorithms*, *Journal of Computer and System Sciences*, vol. 42 (1991), pp. 30–53.

THOMAS HOFWEBER, *Numbers and number determiners*.
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To understand arithmetic one has to understand what the relation is between "two" in the following:

1. Three plus two is five.
2. Two apples are on the table.
3. The number of apples on the table is two.

It seems clear that they have to have some close relation, but, on reflection, it is rather unclear what this relation is. There are a number of substantial differences between them. For example, "two" in (1) and in (3) seem to be of low type, the type of objects (e), whereas "two" in (2) seems to be of high type, the type of determiners $((e,t),((e,t),),t))$.

A number of proposals about their relation have been made (Field, Hodes, Wright and others). However they either are in conflict with insights from natural language semantics, or they overemphasize that number determiners are first order definable, or they introduce an implausibly big gap between them.

In this talk we will outline a new account of the relation between numbers in arithmetic and number determiners in natural language. We will take recourse to evidence from the process of how number concepts are acquired and how basic arithmetic truths are learned. Of particular importance will be the uses of bare determiners (determiners without explicit first argument), both in the plural and the singular.

LEON HORSTEN, *On proposed formalizations of Church's thesis in the language of epistemic arithmetic*.
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This paper evaluates various proposals to formalize Church's Thesis (CT) in the language of Epistemic Arithmetic.

It is first argued that CT cannot be expressed in a satisfactory manner in the language of *first-order* Epistemic Arithmetic. In particular, both Myhill's ([3]) and Flagg's ([1]) proposals for formalizing CT are inadequate. My critique on these proposals is independent of the fact that CT appears at least prima facie to be a second-order statement.

Next I discuss attempts to formalize CT in the *higher-order* language of Epistemic Type Theory ([2]). It is not completely clear from [2] how the language of Epistemic Type Theory ought to be interpreted. In particular, it is unclear what the higher-order quantifiers are supposed to range over. I propose that they are best understood as ranging over *functions* (*or sets*), *given under a presentation*. With the intended interpretation of the language fixed in this way, it becomes clear that even Flagg's own proposal for giving a higher-order formalization of CT is unsatisfactory. But there is a modification of this proposal for which it appears more difficult to establish on either philosophical or logical grounds that it is inadequate.

[1] R. FLAGG, *Church's thesis is consistent with epistemic arithmetic*, *Intensional mathematics* (S. Shapiro, editor), North-Holland, Amsterdam, 1985.

[2] R. FLAGG, *Integrating classical and intuitionistic type theory*, *Annals of Pure and Applied Logic*, vol. 32 (1986).

[3] J. MYHILL, *Intensional set theory*, *Intensional mathematics* (S. Shapiro, editor), North-Holland, Amsterdam, 1985.

RICHARD TIESZEN, *Gödel and the intuition of concepts*.
From January 1, 1998 to June 15, 1998: Department of Philosophy, Stanford University, Stanford, CA 94305, USA.
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Gödel has argued that we can cultivate the intuition of (abstract) concepts in mathematics and logic. The idea that we intuit concepts is related to the following important philosophical themes in Gödel's writings: It is by virtue of the intuition of concepts that we are able to set up and consistently extend formal or mechanical systems in mathematics and logic. The idea is related to the incompleteness of mathematics because we cannot expect to fully capture abstract mathematical concepts in finitely axiomatizable formal systems. This in turn is part of Gödel's realism. The second incompleteness theorem shows that finding consistency proofs will require the development of insight into abstract concepts. The idea of deciding undecidable mathematical propositions by ascending to higher types or by extending mathematical theories with new axioms is best described as developing our intuition of concepts, and it is because we intuit mathematical concepts that we are not forced to conclude that there are absolutely undecidable diophantine propositions. Indeed, the idea that we intuit concepts is related to Gödel's view that there may be 'absolute' notions of provability and definablity. It is because human minds intuit concepts that minds are not machines. In addition, Gödel condemns those forms of reductionism that seek to eliminate the idea of intuition of concepts.

In this talk I claim that the idea of intuiting mathematical concepts is not absurd provided it is understood correctly. I sketch an account of what it means to say that there is an intuition of concepts and I point out why this idea is important for understanding the nature of logic and mathematics.

Abstracts presented by title

BILLY JOE LUCAS, *The logic of time*: *Some questions about tenses*.
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It is useful to look at temporal logic from two different perspectives: as providing a logic of time, and as providing a tense logic. The first part of the talk will focus on issues about the grammar of tense in English, and raises some questions regarding adequacy conditions for a logic of time.

The remainder of the talk addresses basic issues about tense from the point of view of a logic of time. Particular attention is given to the relative merits of proper subsystems of linear time in which convergence replaces "no forks". A nice result is a proof that Hamblin's fourteen tenses theorem holds for transitive, dense time without endpoints when convergence replaces no forks.

CYRUS F. NOURANI, *Admissible models and Peano arithmetic*.
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Admissible computable sets and models were brought forth by [1, 2]. Generic diagrams were applied to define admissible computable sets and models. We had defined computable Functors. The Functors define generic sets from language strings to form limits and models. Nourani 1980's ASL and AMS papers had stated and proved theorems that well-known computational complexity problems were not independent of certain universal fragments of Peano arithmetic. We apply the newer papers [1, 2] to prove:

THEOREM. *There are admissible and L-admissible models* [1, 2] *for Peano Arithmetic generated by functors on infinitary language fragments, at which well-known computational complexity questions are not independent*.

[1] C. F. NOURANI, *Functorial admissible models* (abstract), *Bulletin of Symbolic Logic*, vol. 3 (1997), pp. 390–391.

[2] ———, *Functorial models, admissible sets, and generic rudimentary fragments*, ASL European summer meeting, Leeds, England, July 1997.

DAN E. WILLARD, *The "mirage" effect*.
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Both Roger's textbook [2] and an earlier paper by Kleene [1] show how to apply the Fixed Point Theorem to construct many formalisms α, containing a self-referencing axiom asserting that "*I am consistent*". Such formalisms α are easy to make well defined. However, they are typically inconsistent, despite the presence of the Kleene-like "*I am consistent*" axiom.

We published three articles [3, 4, 5] describing how this difficulty with the "*I am consistent*" axiom can be avoided under some formalisms that can verify all Peano Arithmetic's Π_1 theorems. Our formalisms are called *Self-Justifying Systems*. They view Multiplication (and frequently also Addition) as tertiary relations rather than as total functions.

This talk will announce a new counter-intuitive property of Self-Justifying Systems, called the "*Mirage Effect*". Let the notation $\alpha \vdash \Phi$ and $\alpha \cup \Phi$ retain their conventional meanings of designating α "proves" Φ and $\alpha \cup \Phi$ is the "physical union" of α with the added sentence Φ. If α is a Self-Justifying system, a new notation we will use is $\underline{\alpha} \cup \underline{\Phi}$. It will be called the "*Extensible*" Union of α and Φ. It is slightly stronger than the axiom system $\alpha \cup \Phi$ in that $\underline{\alpha} \cup \underline{\Phi}$ will represent a self-justifying system that differs from α by having its "*I am* *consistent*" axiom reflect the fact that the "I" now refers to an axiom system that includes the new sentence Φ as an axiom. In other words, this means that $\underline{\alpha} \cup \underline{\Phi}$ will differ from α by

1. adding the sentence Φ to α's axioms, and
2. revising α's particular axiom which asserts "*I am consistent*" to reflect Item 1's addition of Φ.

A well known theorem in Logic is that if α is consistent and $\alpha \vdash \Phi$ then $\alpha \cup \Phi$ is also consistent. One might suspect that the preceding two conditions would analogously imply that $\underline{\alpha} \cup \underline{\Phi}$ is also consistent. However, our "*Mirage*" theorem shows that this automatic implication does not follow for the self-justifying systems of our papers [3, 4, 5]. In particular if α denotes any of the self-justifying formalisms from these articles, we can construct a Σ_1 sentence Φ such that $\alpha \vdash \Phi$, $\alpha \cup \Phi$ is sound and consistent, but oddly $\underline{\alpha} \cup \underline{\Phi}$ is inconsistent.

[1] S. Kleene, *On the notation of ordinal numbers*, *Journal of Symbolic Logic*, vol. 3 (1938), pp. 150–156.

[2] H. Rogers, *Theory of recursive functions and effective computation*, McGraw Hill, 1967, pp. 186–188.

[3] D. Willard, *Self-verifying axiom systems*, *Proceedings of the third Kurt Gödel symposium*, Lecture Notes in Computer Science, vol. 713, Springer-Verlag, Berlin, 1993, pp. 325–336.

[4] ———, *The tangibility reflection principle for self-verifying systems*, *Proceedings of the fifth Kurt Gödel symposium*, Lecture Notes in Computer Science, vol. 1289, Springer-Verlag, Berlin, 1997, pp. 319–334.

[5] ———, *Self-reflection principles and NP-hardness*, Dimacs Series in Discrete Mathematics and Theoretical Computer Science, vol. 39, American Mathematical Society, December 1997, pp. 297–320.