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1994 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

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Clermont-Ferrand, France, July 21–30, 1994

The 1994 European Summer Meeting of the Association was held at the Université d'Auvergne in Clermont-Ferrand, France, July 21–30, 1994.

The following main lectures and tutorials were given at the meeting.

Sanjeev Arora (Berkeley, USA), *Probabilistic checking of proofs and hardness of approximation problems*.

Matthias Baaz (Wien-Austria), *Generalization of proofs and term complexity*.

Lenore Blum (Berkeley-USA), *The theory of computation and complexity over the reals*.

Patrick Dehornoy (Caen-France), *The structure group of an algebraic identity*.

Solomon Feferman (Stanford-USA), *How is it that finitary proof theory became infinitary*.

Moti Gitik (Tel-Aviv-Israel), *The singular cardinal hypothesis: the current stage*.

Erich Grädel and Yuri Gurevich (Aachen-Germany), *Finite model theory and beyond*.

Serge Grigorieff (Paris-France), *Decidability and complexity of logical theories*.

David Harel (Rehovot-Israel), *Towards a theory of recursive structures*.

Richard Kaye (Birmingham-UK), *Models of arithmetic: three perpectives*.

Alexander Kechris (Pasadena-USA), *The descriptive set theory of group actions and equivalence relations*.

Daniel Leivant (Bloomington-USA), *Transfer principles between logic-based approaches to computational complexity*.

Maciej Liśkiewicz and Rüdiger Reischuk (Wrocław-Poland and Darmstadt-Germany), *Lower bounds, hierarchies and closure properties for sublogarithmic space classes*.

H. Dugald Macpherson (London-UK), *Variations of o-minimality*.

Larisa Maksimova (Novosibirsk-Russia), *On some aspects of the history of modal logics*.

Bruno Poizat (Lyon-France), *Géométrie des modèles et théorie algébrique.*.

Jean Pierre Ressayre (Paris-France), *Finitistic applications of transfinite power series*.

Simona Ronchi della Rocca and Luca Roversi (Torino-Italy and Pisa-Italy), *Lambda calculus and intuitionistic linear logic*.

Jerzy Tiuryn (Warsaw-Poland), *Simply typed lambda calculus with subtyping – the problem of type reconstruction*.

Johan Van Benthem (Amsterdam-Netherlands), *Modal logics and process theories*.

Moshe Y. Vardi (Houston-USA), *Infinitary logic for computer science*.

Boban Velickovic (North-York-Canada), *Forcing and* $0^\#$.

Victor Vianu (INRIA-Rocquencourt-France), *Relational complexity*.

Frank Wagner (Oxford-UK), *On the structure of stable groups*.

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The abstracts of the main lectures, tutorials, and contributed talks given by members of the Association for Symbolic Logic follow.

For the Program Committee
DENIS RICHARD

Main lectures and tutorials

► SANJEEV ARORA, *Probabilistic checking of proofs and hardness of approximation problems*.
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The classical theory of NP-completeness, pioneered by Cook, Karp and Levin in the 70s, explains the apparent computational intractability of optimization problems by proving them NP-hard. But its techniques were unable to explain why for many optimization problems, even finding good approximation algorithms is difficult (or at least, why significant amounts of research failed to find such algorithms).

Recent work has yielded a simple explanation: many NP-hard optimization problems are also NP-hard to approximate. Among the problems for which this has been proved are Independent Set, Chromatic Number, Set Cover, Vertex Cover, Max-Cut, Decoding to the nearest codeword, Learning in the presence of Errors, and many others. At the heart of these new results is a new type of NP-completeness reduction which acts globally (as opposed to the classical reductions, which used local transformations involving gadgets etc.). A more interesting way to view this reduction is as a new probabilistic definition of NP: NP is exactly the class of languages for which membership proofs (i.e., certificates) can be checked in polynomial time by using only $O(\log n)$ random bits and examining only $O(1)$ bits in them.

Of further interest to mathematicians will be the result that every mathematical proof can be rewritten in a form in which its correctness can be checked (probabilistically) by examining a constant number of bits it it.

The talk will be a survey of this area. It will be self-contained.

► MATTHIAS BAAZ, *Generalization of proofs and term complexity*.
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We investigate the concept of *short proofs*, i.e., proofs of theorems in a fixed number of steps containing complex terms, in various circumstances, namely relative to different axiom systems and to different formulations of the underlying (classical) logical system. Results about generalizations provide a way to distinguish between different proof systems for one and the same theory, i.e., to distinguish between formulations which are indistinguishable by model theoretic properties.

Usually, generalizations of *theorems* are considered, e.g., *Kreisel's Conjecture:* If there is a k s.t. $A(s^n(0))$ is provable in k steps in Peano Arithmetic, then $(\forall x)A(x)$ is provable. The important aspect, however, is generalization of *proofs* and the circumstances and conditions under which it is possible. Generalization of theorems, then, can be viewed as a borderline case of generalization of theorems, where every sound proof transformation is allowed.

The strongest kind of generalization is the calculation of a *term basis* (cf. [1]):

DEFINITION 1. A finite set of n-tuples of terms $B = \{\langle t_1^i, \ldots, t_n^i \rangle\}_{i=1}^m$ is called a *term basis* for $A(x_1, \dots, x_n)$ and a class of proofs Π in a theory T if

(1) $T \vdash A(t_1^i, \dots, t_n^i)$ for $1 \leq i \leq m$,

(2) if $T \overset{\Pi}{\vdash} A(s_1, \dots, s_n)$ (s_j variable free) then there is a substitution σ s.t. for some i ($1 \leq i \leq m$) it holds that $s_j = t_j^i\sigma$ for all j, $1 \leq j \leq n$.

If a formal system has term bases relative to proofs of bounded length, then Kreisel's Conjecture follows from number-theoretic axioms.

Term basis exists, e.g., for Π the class of cut-free proofs of bounded length in a finitely axiomatized theory T. It also exists for the class of all proofs of bounded length in T, if either cut-elimination, or replacement of quantifier inferences by inferences acting on *blocks* of like quantifiers in proof descriptions, is an allowed transformation.

The computation of term bases usually proceeds by obtaining a term-minimal proof, e.g., by defining a unification problem over the skeleton of a given proof. This method works for cut-free **LK**, and we show that it also works for general $\mathbf{LK}_\mathrm{B}$, a variant of **LK** where quantifier inferences introduce sequences of like quantifiers in one step (cf. [2]); it does not work for general **LK**. If we start from a theory T plus the schema S, and use $\mathbf{LK}_\mathrm{B}$ instead of **LK**, the construction of the term-minimal proof transforms single quantifiers in the instances of S into blocks of quantifiers. If these instances are themselves derivable from $T + S$, then term bases exist. This criterion is our main result.

We point out that this criterion holds, e.g., for the schemata of order induction and elementary Dedekind cuts, but of course not for the schema of identity (A result of Yukami shows that there can be no term basis for the schema of identitiy w.r.t. to proofs of bounded length). This also indicates that the character of these results is in no way model theoretic, and hence "genuinely" proof theoretic. Here, the schema of identity is "difficult" while order induction is "easy."

[1] M. Baaz and P. Pudlák, *Kreisel's conjecture for $L\exists_1$*, ***Arithmetic, proof theory and computational complexity*** (P. Clote and J. Krajíček, editors), Oxford University Press, 1993, pp. 29–59.

[2] M. Baaz and R. Zach, *Algorithmic structuring of cut-free proofs*, ***Computer Science Logic. Selected papers from CSL '92*** (E. Börger, editor), LNCS, Springer, Berlin, 1993, pp. 29–42.

► LENORE BLUM, *A theory of computation and complexity over the reals*.

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The theory of computation originated in the 1930's with work of logicians interested in resolving questions of decidability and undecidability. This theory was refined and further developed in the 1960's by computer scientists interested in understanding the intrinsic difficulty—or complexity—of solving (solvable) problems.

The classical theory deals with discrete problems, for example over the integers, about graphs, etc. On the other hand, most computational problems that arise in numerical analysis and scientific computation, in optimization theory and more recently in robotics and computational geometry, have as natural domains the reals, or complex numbers. Adaptations of the classical theory to study these problems are problematic.

In 1989, Blum, Shub and Smale introduced a theory of computation and complexity (over a ring) that integrates key ideas from the classical theory in a setting more amenable to problems defined over continuous domains. The theory yields results in the continuous setting analogous to the pivotal classical results over the integers of Gödel (on undecidability) and Cook (on NP-completeness), yet reflecting the special mathematical character of the underlying space. For example, we have that:

(1) the Mandelbrot set as well as most Julia sets are undecidable over the reals, and
(2) the Hilbert Nullstellensatz is NP-complete over the complex numbers.

I will discuss these results as well as recent developments and directions in the field.

► PATRICK DEHORNOY, *The structure group of an algebraic identity*.
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We propose to associate with every algebraic identity an operator monoid and, in good cases, a derived group, which capture the geometry of that particular identity and may be used to describe the free corresponding objects.

We first concentrate on the case of left selfdistributivity for which the approach was originally created. The decidability of the word problem for free left distributive structures has been proved in 1989 by Richard Laver using the hypothesis that there exists an n-huge cardinal for every integer n. It has been subsequently established by the author without any set-theoretical assumption. The problem is as follows. Let us call a set equipped with a left (self)distributive law an LD-system. Purely algebraic developments had shown that a sufficient condition for the above word problem to be decidable is the existence of an LD-system satisfying an additional algebraic condition known as irreflexivity, or acyclicity. Then large cardinal hypotheses provide elementary embeddings of (fragments of) models of ZFC from which one easily defines an LD-system which is acyclic, and this is the way large cardinals enter the picture. In order to drop the logical hypothesis, it suffices to construct another example of acyclic LD-system. Unfortunately all standard examples of LD-systems are very far from acyclic, and we are left with the question of constructing new distributive operations, and, because an acyclic LD-system must include free LD-systems with one generator, with the more particular question of defining 'concrete' realizations of the free LD-systems.

Since there are no relevant examples to start with, we try to construct a 'syntactic' LD-system whose domain is (a quotient of) the family of all identities which are consequences of LD, an approach which is reminiscent of Henkin's completeness proof for first-order logic where one constructs a model for a given coherent set of sentences whose domain is (a quotient of) the family of all (existential) sentences. In the present case we observe that two terms say P, Q are equivalent up to LD if and only if the pair (P, Q) is an instance of some consequence of LD which can be taken in a fixed canonical family, if and only if some convenient operator maps P to Q. It is easy to define a monoid structure on these operators using composition (or on the identities using unification), thus obtaining a 'structure monoid' $\mathcal{M}_{\mathrm{LD}}$ for the distributivity identity LD. Technically it is convenient to guess a presentation for this monoid and then to work with the group $\mathcal{G}_{\mathrm{LD}}$ which admits this presentation.

The main idea in order to construct a realization of the free LD-system is to associate with every term P a particular operator $\widetilde{P}$ which *constructs* P from some uniform initial term (this is the specific point where the considered identity is really important). The way the operator $\widetilde{P.Q}$ is constructed from $\widetilde{P}$ and $\widetilde{Q}$ gives the receipt for defining an LD-system on the right cosets of $\mathcal{G}_{\mathrm{LD}}$ associated with a certain subgroup H_0, and the proof that this LD-system is acyclic follows from a long but natural study of the group $\mathcal{G}_{\mathrm{LD}}$ using the particular form of its presentation. And because the group $\mathcal{G}_{\mathrm{LD}}$ happens to be an extension of Artin's braid group B_∞ and the associated kernel includes the subgroup H_0, the above LD-system projects to braids. One shows that this projection is injective, so that finally one has obtained a realization of the free LD-system with one generator inside the braid group B_∞ (from this point on one can subsequently deduce new properties of the braids, like the existence of a linear ordering).

Leaving apart the last points which are specific of the case of left distributivity, we think that the above approach can be uniformly applied to a number of algebraic identities. We shall briefly mention the cases of associativity, where the study is easy and gives a new strong form of the so-called MacLane-Stasheff pentagon property, and the case of left distributivity with idempotence where the method could possibly be used to approach recent conjectures by Larue and Kepka.

► SOLOMON FEFERMAN, *How is it that finitary proof theory became infinitary?*
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Proof theory was the technical area of logic instituted by Hilbert in the 1920s in order to carry out his program to establish, by finitary methods, the consistency of axiomatic systems underlying actual mathematics. Hilbert believed that a successful prosecution of his program for number theory, analysis and, eventually, set theory, was necessary in order to justify all manifestations of the infinite (both implicit and explicit) in mathematics. Whatever its merits, Gödel's incompleteness theorems apparently undermined Hilbert's Program as originally conceived; "apparently", because the exact extent of finitary methods were never spelled out, but only understood informally. Nevertheless, the goal to establish the consistency of number theory and stronger systems continued to be pursued, but increasingly with the infusion of infinitary concepts and methods. The first step in that direction was taken by Gentzen in 1936, who used transfinite induction up to the first epsilon number in order to establish the consistency of Peano Arithmetic.

Beginning in the 1950s, proof theory began to employ infinitary methods more frankly, first with the use of infinitary rules of inference (such as the omega rule) and then with the use of infinitely long formulas. At first, these rules, formulas and the resulting derivations were all countable, but eventually proof theory moved on to make use of prima facie uncountably long rules of inference and/or formulas, thence uncountably long derivations. These countably and uncountably long derivations have been applied to treat various formal systems for parts of analysis and set theory, by translating their finite derivations into corresponding infinite ones of a more purely logical character. By then applying the process of cut-elimination or other forms of normalization, the latter are transformed into detour-free derivations whose associated ordinals give a measure of complexity to the systems thus treated. For example, they can be used to classify their provably recursive functions. Cut-free derivations are easily shown not to end in a contradiction, so one also concludes consistency from such analyses.

In general, the more that such infinitary methods were employed, the farther did proof theory depart from its initial aims and methods, and the closer did it come instead to ongoing developments in recursion theory, particularly as generalized to admissible sets; in both one makes use of analogues of regular cardinals, as well as "large" cardinals (inaccessible, Mahlo, etc.). By reference to some chapters at different points in this transformation of our subject, these lectures will explore how and why these remarkable changes took place. We will also take up the question of finding a new and more appropriate rationale for proof theory, given its (all but) whole-hearted embrace of infinitary methods.

Where does that leave finitary proof theory? If time permits, there will be a brief introduction to its unexpected successes in recent years, disjoint from all of the above.

[1] P. ACZEL, H. SIMMONS, and S. WAINER, ***Proof theory***, Cambridge University Press, 1992.

[2] J. BARWISE (editor), ***Handbook of mathematical logic***, North Holland, 1977.

[3] W. BUCHHOLZ, S. FEFERMAN, W. POHLERS, and W. SIEG, ***Iterated inductive definitions and subsystems of analysis: Recent proof-theoretical studies***, Lecture Notes in Mathematics, no. 897, Springer, 1981.

[4] W. BUCHHOLZ and K. SCHÜTTE, ***Proof theory of impredicative systems***, Bibliopolis, 1977.

[5] S. FEFERMAN, ***Lectures on proof theory***, Lecture Notes in Mathematics, no. 70, Springer, 1968.

[6] G. GENTZEN, ***The collected papers of Gerhard Gentzen***, North Holland, 1969.

[7] J. Y. GIRARD, ***Proof theory and logical complexity***, Bibliopolis, 1987.

[8] D. HILBERT and P. BERNAYS, ***Grundlagen der Mathematik***, 2nd ed., vol. 1–2, Springer, 1968.

[9] G. JÄGER, ***Theories for admissible sets. A unifying approach to proof theory***, Bibliopolis, 1986.

[10] G. E. MINTS, ***Selected papers in proof theory***, Bibliopolis, 1992.

[11] J. MYHILL, A. KINO, and R. E. VESLEY (editors), ***Intuitionism and proof theory***, North Holland, 1970.

[12] W. POHLERS, ***Proof theory, an introduction***, Lectures Notes in Mathematics, no. 1407, Springer, 1989.

[13] K. SCHÜTTE, ***Proof theory***, 2nd ed., North Holland, 1977.

► MOTI GITIK, *The singular cardinal hypothesis: the current stage*.
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We describe the current stage of the problem. The particular emphasis will be devoted to recent forcing constructions for blowing up the power of a singular cardinal.

► ERICH GRÄDEL AND YURI GUREVICH, *Metafinite model theory*.
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Although questions involving finite structures have always been of interest to logicians, finite model theory has emerged as a separate research area only in the 1970's. Part of the motivation came from computer science, in particular from databases and complexity theory. Finite structures posed a nontrivial challenge for mathematical logic, in particular for model theory, since most of the important classical results and techniques of mathematical logic (such as, e.g., compactness, completeness, the usual preservation theorems) fail when only finite structures are considered.

Even though we believe that finite model theory has been rather successful, the time has come to re-examine the situation. Motivated again by challenges from computer science we feel that the strict adherence to finiteness is too restrictive and suggest to *extend the approach and methods of finite model theory beyond finite models*.

We propose a more general class of structures, which we call *metafinite structures* and a number of logics to reason about them. Typical metafinite structures consist of **(i)** a primary part, which is a finite structure, **(ii)** a secondary part, which may be finite or infinite, and **(iii)** a set of "weight" functions from the first part into the second. However, the infinity that we seek is very modest. It should not manifest itself too obtrusively, deviating our attention to phenomena that are pertinent to infinite structures only. Therefore our logics of metafinite structures—appropriate modifications of the usual logics of interest to finite model theory, such as first-order logic, fixed point logics or $L^{\omega}_{\infty\omega}$—access the infinite part only in a limited way. An important feature of these logics is that they contain, besides formulae and terms in the usual sense, a calculus of functions from the primary to the secondary part, which we call *weights*.

We investigate the expressive power of these logics and reconsider issues of finite model theory, like descriptive complexity, asymptotic probabilities, back and forth systems, in this extended framework. In many cases the metafinite framework turns out to be more natural and convenient.

Applications of metafinite model theory arise, for example, in databases, optimization, complexity theory and discrete dynamic systems. In particular, metafinite model theory provides a proper setting for a descriptive complexity theory over the real numbers with respect to the model of Blum, Shub and Smale.

► SERGE GRIGORIEFF, *Decidability and complexity of logical theories*.
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The *first talk* will present the decision problem in a historical perspective and give the most salient results obtained in the last 60 years:

- its emergence from Leibniz to Frege,
- first positive results
  - propositional calculus : Post, 1921;
  - fragments of predicate calculus : Bernays & Schonfinkel, 1928, Ackermann, 1928, Gödel, 1930;
  - real algebra : Tarski, 1931,
  - additive number theory : Presburger, 1930,
- the general negative solution
  - predicate calculus and number theory : Church, Turing, 1936, based on Gödel, 1931),
- subsequent positive results
  - boolean algebras : Tarski, 1949,
  - abelian groups : Szmielew, 1949,
  - monadic theory of one successor : Büchi, 1962;
  - case of two successors : Doner, 1965, Rabin, 1969
- the intrinsic complexity results (Meyer, 1972; Fischer & Rabin, 1974).

In the *second talk*, methods will be presented to obtain decidability and upper complexity bounds.

The *last talk* will be devoted to undecidability and lower bounds.

[1] E. Börger, *Decision problems in predicate logic*, ***Logic Colloquium '82***, North-Holland, 1984, pp. 263–301.

[2] K. Compton and C. W. Henson, *A uniform method for proving lower bounds on the computational complexity of logical theories*, ***Annals of Pure and Applied Logic***, vol. 48 (1990), pp. 1–79.

[3] B. Dreben and W. Goldfarb, ***The decision problem : solvable classes of quantificational formulas***, Addison-Wesley, 1979.

[4] J. Ferrante and C. Rackhoff, ***The computational complexity of logical theories***, Lecture Notes in Mathematics, no. 718, Springer, 1979.

[5] S. Grigorieff, *Décidabilité et complexité des théories logiques*, ***Logique et informatique: une introduction*** (B. Courcelle, editor), INRIA, 1991, pp. 7–97.

[6] Y. Gurevich, *Monadic theories*, ***Abstract model theory and stronger logics*** (J. Barwise and S. Feferman, editors), Springer, 1984, pp. 479–506.

[7] H. Lewis, ***Unsolvable classes of quantificational formulas***, Addison-Wesley, 1979.

[8] M. O. Rabin, *Decidable theories*, ***Handbook of mathematical logic*** (J. Barwise, editor), North-Holland, 1977, pp. 595–629.

► DAVID HAREL, *Towards a theory of recursive structures*.
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In computer science, one is interested mainly in finite objects. Insofar as *infinite* objects are of interest, they must be computable, i.e., recursive, thus admitting an effective finite representation. This leads to the notion of a recursive graph, or, more generally, a recursive structure or data base. In this paper we summarize our recent work on recursive structures and data bases, including

(i) the high undecidability of many problems on recursive graphs,

(ii) somewhat surprising ways of deducing results on the classification of NP optimization problems from results on the degree of undecidability of their infinitary analogues, and

(iii) completeness results for query languages on recursive data bases.

[1] D. Harel, *Towards a theory of recursive structure*, ***Proceedings of the Symposium on Theoretical Aspects of Computer Science***, Lecture Notes in Computer Science, Springer-Verlag, 1994.

► RICHARD KAYE, *Models of arithmetic: three perspectives*.
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Research in model theory of first-order Peano Arithmetic and its subtheories can be loosely divided into three main areas: 1. "algebraic" models; 2. models of "weak" theories and complexity theory; and 3. "strong" theories and their models. There are some connections between these areas and the dividing lines are not always so well defined, but each area has its own distinctive flavour.

Each of my three talks will be devoted to one of these areas. My intention is primarily to give the listeners a good idea of the type of work that is currently being carried out in each area (and necessarily some relevant older results), and to illustrate the flavour of the subjects by giving at least one sketch proof of an important result in each area. Time will not permit full proofs, nor a comprehensive survey of any of these topics, but I hope to whet the listeners' appetite for further investigation.

The three talks will each be mostly self-contained and will require the minimum of prior knowledge. They consist of

1. "Algebraic" models: Shepherdson's characterization of models of open induction as models consisting of integral parts of elements of their real closure. Wilkie's results. Van den Dries' results and more recent work on normal models of open induction.

2. Models of "weak" theories and complexity theory: The model-theoretic attack on complexity theory; the theories $I\Delta_0$ and $I\Delta_0 + \exp$; Parikh's theorem; Buss' hierarchy of theories, and the Krajíček–Pudlák–Takeuti result connecting this hierarchy with the polynomial-time hierarchy; relativizations of the hierarchy problems.

3. "Strong" theories: the theories $I\Sigma_n$, $B\Sigma_n$, and a rapid overview of "local" results in PA and its fragments; "nonlocal" results, including results by Gaifman and others; recursively saturated models and their structural properties.

► ALEXANDER KECHRIS, *The descriptive set theory of group actions and equivalence relations*.
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Descriptive set theory is the study of definable sets in Polish spaces. Although these include many of the spaces studied in mathematics, there are also spaces of fundamental importance which are obtained instead by starting with a Polish space X and a definable equivalence relation E on X and considering the quotient space X/E.

Examples include the space of orbits of a Borel automorphism or more generally of a Borel action of a Polish group, the space of countable models of a theory modulo elementary equivalence or isomorphism, the space of Turing degrees, etc.

In these talks, I will discuss some recent progress in the study and classification of definable equivalence relations on Polish spaces and their quotient spaces.

► DANIEL LEIVANT, *Transfer principles between logic-based approaches to computational complexity*.

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Machine independent approaches to computational complexity establish the importance of certain classes, provide insight into their nature, relate them to issues relevant to programming methodology and to program verification, suggest new separation tools, and offer concepts and methods for generalizing computational complexity to computing over arbitrary structures and to higher type functionals.

Machine independent characterizations fall, by and large, into three major groups: database queries (methods of finite model theory), applicative programs over free algebras (bounded recurrence, tiered recurrence), and proof-theoretic (restricted comprehension, bounded induction, tiered induction).

Towards a unified theory of computational complexity, we describe transfer principles that link various approaches, and in so doing we derive some new results.

► MACIEJ LIŚKIEWICZ AND RÜDIGER REISCHUK, *Lower bounds, hierarchies and closure properties, for sublogarithmic space classes*.

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We try to fully characterize the properties and relationships of space classes defined by Turing machines that use less than logarithmic space - may they be deterministic, nondeterministic or alternating. Because of the limited counting ability of such machines most of the standard simulation techniques do not work for sublogarithmic space classes. However, machines with such little space may still be quite powerful. Therefore, it was not obvious how to obtain analogs for inclusion and separation results known for classes above logspace.

We provide several examples of specific languages and show that machines using sublogarithmic space are unable to accept these languages. These lower bounds show that the complexity world below logarithmic space really behave differently. For example, certain closure properties do not hold. The restricted power of these machines makes it possible to prove explicit separations—even for alternating complexity classes—by combinatorial arguments and to obtain a hierarchy of non-relativized complexity classes without any unproven assumption.

Finally, these complexity classes are related to other classes within P, in particular to context-free languages.

► H. DUGALD MACPHERSON, *Variations of o-minimality*.
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One way of defining o-minimality is to say that a structure $\mathcal{M} = (M, <, \dots)$ is *o-minimal* if $(M, <)$ is totally ordered and, for every $\mathcal{N}$ elementarily equivalent to $\mathcal{M}$, every definable subset of N is quantifier-free definable by a formula just involving $<$ (and equality) (in fact, for o-minimality one doesn't need to mention $\mathcal{N}$). The talk concerns attempts to vary this definition by replacing $<$ with various other relation symbols (with corresponding axioms in place of those of a total order). I will describe joint work with Steinhorn on various such minimality notions, work with Haskell on notions of minimality (C-minimality and P-minimality) particularly appropriate for algebraically closed valued fields and for the p-adics, and joint work with Marker and Steinhorn on another related notion, weak o-minimality, appropriate for real closed convexly valued fields and related structures.

The theorems obtained take a similar form in each case. Any weakly o-minimal ordered field is real closed, any C-minimal field is an algebraically closed field with a definable valuation, and any P-minimal field is p-adically closed. A key ingredient in these results is topological dimension. All these structures carry a topology with a uniformly definable basis. We define the dimension of any (parameter)-definable subset X of M^n to be the maximal r such that for some projection $\pi : M^n \longrightarrow M^r$, $\pi(X)$ has non-empty interior in M^r. In each case, it is shown that if $X_1, \dots, X_t$ are definable subsets of M^n then $\dim(X_1 \cup \dots \cup X_t) = \mathrm{Max}\{\dim(X_1), \dots, \dim(X_t)\}$. Simultaneously, continuity results are proved for definable functions $F^n \longrightarrow F$, and in some cases there are cell decomposition theorems. There are also characterizations of when algebraic closure has the exchange property (if it does, it gives the same notion of dimension).

In the talk, I will survey the relevant background from o-minimality. I will then describe the various new notions of minimality and their motivating examples, and discuss the main theorems.

► LARISA MAKSIMOVA, *On some aspects of the history of modal logics*.
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(1) First formal systems of modal logics (Lewis 1918, Lewis & Langford 1932). Modalities and strict implication. Various axiomatizations of modal systems.
(2) Applying modalities for interpretation of the intuitionistic logic (Gödel 1933). Translation of the intuitionistic logic into the modal system S4. Algebraic and topological semantics of S4 constructed by McKinsey and Tarski.
(3) From study of individual logical calculi to the research of families of propositional logics. The logic as a set of its theorems. Intermediate logics (Umezawa 1955, 1959) and their translation into modal logics between S4 and S5 (Dummett & Lemmon 1959). Universality of the algebraic semantics. The cardinality of the families of intermediate and of modal logics (Yankov 1968).
(4) Relational semantics of modal logics (Kripke 1959, 1963, Hintikka 1961, Kanger 1957). Normal and non-normal modal systems. Interrelations of algebraic and relational semantics. Representation theorems for Boolean algebras with operators (Jonsson & Tarski 1951).
(5) Investigation of Kripke-completeness of propositional modal systems. Deleting the reflexivity axiom. Examples of non-complete modal logics (with respect to the relational semantics) (Fine 1974, Thomason 1974). Generalized Kripke semantics

(Makinson 1970). Neighborhood semantics (Montague 1970) and topological semantics. Completeness and non-completeness.

(6) Decidability and non-decidability in propositional modal logics. Finite model property. Filtration. Gentzen formulations of some modal systems. Applying Rubin's theorem. Examples of non-decidable propositional modal calculi (Thomason 1975).

(7) Quantifiers in modal logics (Barcan 1946). Barcan Formula.

► BRUNO POIZAT, *Théorie des modèles et géométrie algébrique*.
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Cette conférence est consacrée à la convergence de la Théorie des modèles vers la Géométrie algébrique qu'on observe depuis quelques années. Bien des mathématiciens, surtout dans ce pays, trouveront contre-nature ce rapprochement entre la Logique et une manifestation si prestigieuse de l'activité scientifique ; cependant, si on le voit avec quelque recul, il semble inévitable : l'essentiel de l'étude des géomètres a pour sujet des êtres définissables dans un corps algébriquement clos, si bien qu'on considérerait la Géomètrie algébrique comme la partie de la Théorie des modèles consacrée aux corps si cette étude n'était pas plus fine que celle qu'en font les logiciens. Ces derniers, par nécessité, ont dû chercher à comprendre les travaux des géomètres quand ils ont vu apparaître, dans le contexte abstrait de la classification des structures, des objets de même comportement que ceux de la Géométrie. En plusieurs occasions, ils se sont même demandé s'ils ne leur étaient pas identiques.

Un exemple d'une telle situation est la récente caractérisation abstraite, par Hrushovski et Zil'ber, d'une courbe lisse à partir de la topologie de Zariski de ses puissances cartésiennes.

Cette caractérisation est lourdement chargée d'histoire, étant l'aboutissement d'un long processus modèle-théorique. Mais c'est aussi la promesse d'un avenir où les géomètres manifesteront un intérêt réciproque pour la Théorie des modèles. En effet Hrushovski a déduit de cette caractérisation la première preuve complète, en toute caractéristique, de la conjecture dite de Mordell-Lang à propos des sous-groupes de type fini d'une variété abélienne. C'est à mon avis la première application significative de la Logique aux Mathématiques classiques.

Cette preuve a la propriété particulièrement intéressante d'utiliser une structure auxilliaire - un corps différentiellement clos en caractéristique nulle, un corps séparablement clos en caractéristique p - pour aboutir à un énoncé qui ne fait intervenir que la notion de corps. Elle vient de la maîtrise des propriétés modèle-théoriques de cette structure auxilliaire, dont on doit en particulier montrer que des fragments substantiels ressemblent aux courbes lisses dont il a été question plus haut. Vu sous cet angle, Mordell-Lang apparaît comme la partie émergeante d'un iceberg, comme une espèce de traduction superficielle de résultats structurels profondément engloutis.

Je dirai quelques mots plus précis sur les raisons qui ont mené les théoriciens des modèles à la Géométrie, j'essayerai d'expliquer en quoi consiste l'énoncé de Mordell-Lang - ce n'est pas un exercice si facile devant un auditoire peu familier du sujet - et je décrirai les grandes lignes de la démonstration.

► JEAN PIERRE RESSAYRE, *Finitistic applications of transfinite power series*.
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In the forties, Malcev and B. Neumann proved:

(0) every real closed field K is isomorphic to some field K' of **transfinite** power series (with the operations induced by the structure of these series).

In the nineties, three extensions and complements were obtained (see [1, 3, 2]):

(1) ensuring that K' is **truncation closed** (for every series $S \in K'$ of ordinal length α and for every $\beta < \alpha$, the sum of the first β terms of S also belongs to K'). This is important in the proof of (2), (3) below.
(2) extending (0) to the case where K, K' are models of the complete theory of ($\mathbb{R}, e^x$ restricted to $[0, 1]$). (or of other expansions of the field $\mathbb{R}$ by restricted analytic functions).
(3) constructing in every real closed field K an "**Integral Part**", that is a subring A providing every element x of K with a unique "integer part" $[x] \in A$ such that $[x] \leq x < [x] + 1$. Moreover, if K is equipped with a function x^y satisfying the basic properties of this function over $\mathbb{R}$, then this Integral Part A can be chosen so that A^+ is closed under x^y.

The author applied (1) (2) (3) to questions of an effective nature: his results concern the polynomial time computability of certain functions over $\mathbb{N}$ or $\mathbb{R}$; and eliminate complicated schemes from the recursive axiomatization of $(\mathbb{R}, e^x)$ provided by Macintyre-Wilkie (who prove its completeness under Shanuel's Conjecture). For most of these effective results, only the proof based on (1) (2) (3) exists, and the problem of finding a less exotic, more constructive proof seems to be a challenge.

[1] M. H. MOURGUES and J. P. RESSAYRE, *Every real closed field has an integer part*, ***The Journal of Symbolic Logic***, vol. 52 (1993), pp. 641–647.

[2] J. P. RESSAYRE, *Integer parts of real closed exponential fields*, ***Proof theory, arithmetic and computational, complexity. Proceedings of the Prague 1991 workshop*** (P. Clote and J. Krajicek, editors).

[3] L. VAN DEN DRIES, A. MACINTYRE, and D. MARKER, *The elementary theory of restricted analytic fields with exponentiation*, ***Annals of Mathematics***, vol. 140 (1994), pp. 183–205.

► SIMONA RONCHI DELLA ROCCA AND LUCA ROVERSI, *Lambda calculus and intuitionistic linear logic*.
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The *Curry-Howard isomorphism*, or "formulas-as-types" principle, can be seen as the basis of typed functional programming. In fact, through this isomorphism, the intuitionistic proof of a formula can be seen as a functional program, whose type is the formula itself. Then the computation process has its logic correspondence in the proof normalization procedure, or "cut elimination". Well known examples of such an isomorphism are the implicational fragment of the intuitionistic propositional logic and the simply typed λ-calculus, or the second order propositional logic and the second order λ-calculus of Girard and Reynolds.

The *linear logic*, introduced by Girard [3], seems to be particularly interesting from the computational point of view. In fact, it is more refined than the classical logic, in the sense that the use of the structural rules is explicitly controlled through a modal connective: more precisely, weakening and contraction rules can be applied only to modal formulas. Since weakening and contraction rules are naturally related to the operations of erasing and copying information, respectively, the linear logic can be seen as a model for a computational environment with an explicit control over the resources management. This features can be effectively studied using a language corresponding to the intuitionistic fragment of the linear logic, through the Curry-Howard isomorphism.

Until now, some languages inspired by this isomorphism have been designed, by Lafont [4], Abramsky [1], Mitchell and Lincoln [5] and Benton, Bierman, de Paiva and Hyland [2].

We propose a new language $\Lambda_!$, representing the intuitionistic linear proofs and having both a simpler syntax and an easier operational semantics than the previous proposed languages. It is well known that the λ-calculus is interesting and useful, as paradigmatic language for the intuitionistic calculus, because of its simplicity. It has just two term formation rules, abstraction and application, and one evaluation rule, the β-rule, which is essentially the substitution. In the same line, a language can be used as a paradigm for the linear intuitionistic computation, only if it preserves both the syntactical and the computational simplicity of the λ-calculus.

The language $\Lambda_!$ is a fully typed language. It is obtained as a "decoration" of the intuitionistic linear proofs, given in natural deduction style. Syntactically, the most peculiar features of $\Lambda_!$ are that terms are built starting from two different sorts of variables, and that not only variables, but also *patterns*, can be bound. A pattern is an expression formed by a logic connective applied to variables (for example, $X \otimes Y$). The two different sorts of variables correspond respectively to modal and non modal premises. The fact that structural rules can be applied only to modal premises reflects that terms must obey to linearity constraints only on one sort of variables. Thanks to the use of patterns, the reduction rules (induced by the cuts of the logics) have a uniform definition: they are all extensions of the classical β-rule, based on pattern-matching. Consequently the computational behavior of $\Lambda_!$ is very simple, and properties of the language, like the strong normalization, for example, can be easily proved by using standard techniques.

Moreover, we define a denotational semantics of $\Lambda_!$ in a categorical setting. This allows the development of general tools for studying operational properties of the language. For example, we prove that the operational equivalence is extensional.

It is possible to define an untyped version of $\Lambda_!$, and a type assignment system giving it linear formulas as types. This is done by means of a function which erases the type information on terms of $\Lambda_!$. The type assignment inherits from $\Lambda_!$ all the good properties, like subject reduction and strong normalization. Further, the principal type property holds for the type assignment, and it is possible to extend it to universal quantified types or intersection types, expanding the class of typable terms.

[1] S. Abramsky, *Computational interpretation of linear logic*, Technical Report 90/92, Department of Computing,Imperial College, London, 1990.

[2] N. Benton, G. Bierman, V. de Paiva, and M. Hyland, *Term assignment for intuitionistic linear logic*, Technical Report 262, Computer Laboratory, University of Cambridge, August 1990.

[3] J. Y. Girard, *Linear logic*, ***Theoretical Computer Science***, vol. 50 (1987), pp. 1–102.

[4] Y. Lafont, *The linear abstract machine*, ***Theoretical Computer Science***, vol. 59 (1988), pp. 157–180.

[5] P. Lincoln and J. Mitchell, *Operational aspects of linear lambda calculus*, ***Proceedings of the 7th Symposium on Logic in Computer Science***, IEEE Computer Society Press (1992), pp. 235–246.

► JERZY TIURYN, *Simply typed lambda calculus with subtyping – the problem of type reconstruction.*

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Types are used in programming languages in order to decrease the possibility of making some kind of run-time errors, such as calling a procedure with incorrect actual parameters. So, from this point of view, types are decorations attached to pieces of programs. Types are very useful in programming but they often tend to be very large, making the program difficult to read. Thus the problem of type reconstruction comes naturally as the problem of deciding for a given program without types whenever it can be correctly typed. This problem is of a certain practical importance since it can be encountered in some programming languages, such as ML—in such languages the type reconstruction is being solved automatically by a compiler.

During the lecture we will discuss the issue of type reconstruction for the simply typed lambda calculus with the relation of subtyping. Sybtyping can be introduced as a partial order on the set of atomic types which is further extended to all simple types. The problem of type reconstruction for the simply typed lambda calculus with subtyping has been the subject of an intensive research lately. We will discuss the main results obtained in this area so far. In particular we will discuss the following topics:

- General upper and lower bounds on the complexity of this problem.
- Partial orders for which the problem is easy.
- Satisfiability of sybtype inequalities in term algebra — this is an algebraic problem (a generalization of the unification problem) to which the type reconstruction problem is reducible in polynomial time.
- The retraction problem for finite partial orders — this is another algebraic problem. The previous problem is exponential time reducible to it.

► JOHAN VAN BENTHEM, *Modal logics and process theories*.
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Dynamic logics in a broad sense are still gaining ground as a way of studying various kinds of process in computer science and recently also cognitive science and linguistics. These logics employ a two-level modal language combining propositions and programs, which is naturally interpreted over "labeled transition systems" (i.e., polymodal Kripke models). A key semantic invariance between such process models has turned out to be "bisimulation", a notion which has emerged independently in modal logic and computer science.

In this setting, there is a variety of operators which may be considered as "logical constants": propositional connectives, program operations, but also many operators moving back and forth between programs and propositions. Our main concern in this lecture will be to get a more systematic view of the natural candidates here, working toward functional completeness theorems that describe natural repertoires of logical constants in dynamic logic. For this purpose, we introduce two notions of "invariance" and "safety" for bisimulation, which allow us to classify all possibilities in the above categories (always modulo this central process equivalence).

Dynamic logic is essentially a marriage between two standard partners: propositional modal logic and a relational algebra for program expressions. Subsequently, we shall look at two ways of extending this framework. The first extension involves so-called "arrow logics" that generalize relational algebra, essentially, making the entire system a two-sorted modal logic. The second generalization involves current "process algebra".

We generalize the earlier notions of invariance and safety to these broader settings, and show how our functional completeness theorems must change. As a side effect of our analysis, we find a novel modal formulation for studying process algebra.

[1] VAN BENTHEM, ***Language in action. categories, lambdas and dynamic logic***, Studies in Logic, no. 130, North-Holland, Amsterdam, 1991.

[2] ———, *Programming operations that are safe for bisimulation*, Report 93-179, Center for the Study of Language and Information, Stanford University, 1993.

[3] ———, *Logic and the flow of information*, ***Proceedings 9th International Congress of Logic, Methodology and Philosophy of Science. Uppsala 1991*** (D. Prawitz, B. Skyrms, and D. Westerstahl, editors), Elsevier Science Publishers, 1994, pp. 693–724.

► MOSHE Y. VARDI, *Infinitary logic for computer science*.
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Infinitary logic $L_{\infty\omega}$ extends first-order logic by allowing infinitary conjunctions and disjunctions (i.e., conjunctions with an infinite number of conjuncts and disjunctions with an infinite number of disjuncts). One usually thinks of infinitary logic as a fairly esoteric logic, which is not of much interest in computer science. Surprisingly, a certain fragment $L^{\omega}_{\infty\omega}$ of $L_{\infty\omega}$ turns out to be of great interest in computer science. This fragment is obtained by restricting formulas to contain a finite number of distinct variables, though the formulas can be of infinite length, The advantage of dealing with $L^{\omega}_{\infty\omega}$ is that its the expressive power can be completely characterized in game-theoretic terms.

We will describe applications of this logic to the study of 0-1 laws and the expressive power of database query languages.

[1] PH. G. KOLAITIS and M. Y. VARDI, *On the expressive power of Datalog: tools and a case study*, ***Proceedings of the 9th ACM symposium on principles of database systems***, 1990, pp. 61–71. Full version to appear in ***Journal of Computer and System Sciences***.

[2] ———, *Fixpoint logic vs. infinitary logic in finite-model theory*, ***Proceedings of the 7th IEEE symposium on logic in computer science***, 1992, pp. 46–57.

[3] ———, *Infinitary logics and 0-1 laws*, ***Information and Computation***, vol. 98 (1992), pp. 258–294.

► BOBAN VELICKOVIC, *Forcing and* $0^{\#}$ *complexity of the reals in inner models of set theory*.
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We present results from a joint paper with Hugh Woodin.

The set of constructible reals is a Σ^1_2 set of reals. This set can have a simpler definition if either it is countable or if every real is constructible. Martin and Solovay showed that it is relatively consistent with ZFC that this set is uncountable and co-analytic.

H. Friedman asked if this set can be analytic or even Borel in a nontrivial way, i.e. both uncountable and not equal to the set of all reals. There is a related problem due to K. Prikry who asked if there can exist a nonconstructible perfect set consisting of constructible reals.

We give a strong negative answer to Friedman's question by showing that if M is an inner model of set theory such that the reals of M form an analytic set then either all reals are in M or $\aleph_1^M$ is countable. The proof relies on a new of regularity property of analytic sets and can be extended to projective sets under the appropriate large cardinal assumptions. However, we show that it is consistent to have an inner model of set theory M such that the reals in M form an uncountable F_σ set. Of course, in this case CH must fail in M.

Concerning Prikry's question we show that it is consistent that there exists an inner model M of set theory which computes correctly $\aleph_1$, there is a perfect set consisting of reals from M, and yet not every real is in M.

We also present some related results in the context of the axiom of determinacy and the failure of the axiom of choice.

► VICTOR VIANU, *Relational complexity*.
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One of the exciting developments in complexity theory is the discovery of a very intimate connection between computational complexity and logic. This intimate connection was first discovered by Fagin, who showed that the complexity class NP coincides with the class of properties expressible in existential 2nd-order logic. Although the relationship between descriptive and computational complexity is intimate, it is not without its problems. Indeed, there is a fundamental mismatch between computational devices and complexity, already noted by mathematicians like Tarski and Harvey Friedman: while computational devices work on *encodings* of problems, logic is applied directly to the underlying mathematical structures. As a result, machines are able to enumerate objects that are logically unordered. For example, while we typically think of the set of nodes in a graph as unordered, it does become ordered when it is encoded on the tape of a Turing machine. This "impedance mismatch" does not pose any difficulty in the identification of NP with existential 2nd-order logic since the logic can simply assert the existence of the desired order. The mismatch with logic becomes apparent, however, at complexity classes below NP. For example, the relationship between the class P and fixpoint logic is more complicated as a result of the mismatch. It is only when we assume built-in order that we get that P coincides with the class of properties expressible in fixpoint logic.

We present a device operating directly on structures, without encoding, called *relational machine*. There is a close connection between relational machine and infinitary logic with finitely many variables. Indeed, relational machines can be viewed as the "effective fragment" of this logic. The relational machine yields a new notion of complexity, quite different from classical computational complexity, which we call *relational complexity*. We show natural correspondences between relational complexity classes and logics, without any order assumption. For example, relational polynomial time corresponds *precisely* to the fixpoint queries, and relational polynomial space yields the while queries. Moreover, relational machines help overcome the "impedance mismatch" between logic and *classical* complexity, by acting as a mediator between logic and classical devices. A typical result of this type is that *fixpoint* = *while* iff P = PSPACE. Similarly, we present logical analogs of the inclusions between complexity classes from P to EXPTIME, in terms of various fixpoint logics.

This work is largely motivated by the study of database query languages and their complexity. Database queries are posed against an abstract interface, so they lead to computations more accurately modeled by relational machines than classical devices. In particular, using the relational machine, we obtain results which point to new possibilities for optimizing queries, and evaluate this using probabilistic techniques related to 0-1 laws. This yields some surprising results on the expected complexity of many queries, which is shown to be lower than the overly pessimistic worst-case complexity.

This talk is based on joint work with Serge Abiteboul, and partly with Kevin Compton and Moshe Vardi.

► FRANK WAGNER, *On the structure of stable groups*.
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In this talk, we shall present results about the group-theoretic properties of stable groups. These can be classified into three main categories, according to the strength of the assumptions needed: chain conditions, generic types, and some form of rank.

Each category has its typical application: chain conditions often allow to deduce global properties from local ones, generic properties are used to get definable groups from undefinable ones, and rank (in some form or other) is necessary to interpret fields in certain group actions.

While originally the main input came from algebraic group theory, founded on the similarities between (finite) rank and dimension, increasingly methods borrowed from finite group theory have come to play their role.

We shall thus look at characteristic subgroups such as the Fitting subgroup, the soluble radical, and the Frattini subgroup, and at characteristic families of subgroups such as p-Sylow subgroups or Carter subgroups.

Finally, we shall propose a conjecture regarding the conjugacy of the Borel subgroups of a group of finite Morley rank.

Abstracts of contributed papers

► A. NAIT ABDALLAH, *The dynamics of logic systems and its applications*.
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Logic is usually conceived as a geometry of static truths. Reasoning about actions in the context of partial information naturally leads to viewing logic as a "dynamics of processes." This view yields a simple solution to a wide range of instances of the *frame problem*: Yale shooting problem, Robot problem, Projection problem, etc.

We use partial information ionic logic, a monotonic framework specifically designed for reasoning with partial or tentative information [1, 2]. Given a set of ionic formulae Ψ, the justification ordering on ionic interpretations $i, i' \in \Delta_\infty$ is defined by: $i \leq_{\star p}^{\Psi} i'$ if and only if i accepts more justifications $\in \Psi$ and rejects less justifications $\in \Psi$ than i'. Let I be some linearly ordered set. Let $\mathcal{F}$ the set of partial information ionic formulae. A syntactic path is a monotone mapping $\Phi : I \rightarrow \mathcal{P}(\mathcal{F}), n \mapsto \Phi_n$ such that $n \leq n'$ implies $\Phi_n \subseteq \Phi_{n'}$. Given a syntactic path Φ, the characteristic surface $\Sigma(\Phi)$ is defined as being the set

$$\Sigma(\Phi) = \prod_{n \in I} (\Omega_{min}(\models)(\Phi_n) / \equiv_{\star}^{G(\cup_n \Phi_n)})$$

where (i) for any set of formulae Ψ, $\Omega_{min}(\models)(\Psi) = \{m \in \Delta_\infty : m \text{ is a } \sqsubseteq\text{-minimal model of } \Psi\}$, where $\sqsubseteq$ is the information ordering ; (ii) $G(\cup_n \Phi_n)$ is the set of formulae having a justification occurrence in some formula of $\cup_n \Phi_n$; and (iii) $\equiv_{\star}^{G(\cup_n \Phi_n)}$ is the equivalence relation generated by the ordering relation $\leq_{\star}^{G(\cup_n \Phi_n)}$, where $(i_0, J, i_1) = i \leq_{\star}^{G(\cup_n \Phi_n)} i' = (i_0', J', i_1')$ if and only if $i \leq_{\star p}^{G(\cup_n \Phi_n)} i'$ and $i_0 \sqsubseteq i_0'$ and $i_0' \sqsubseteq i_1$.

A semantic path for Φ is a $\sqsubseteq$-chain $(m_n)_{n \in I}$ of interpretations such that $\forall n \in I$, m_n is a model of Φ_n, and $\sqcup_n m_n$ is a model of $\cup_n \Phi_n$. A semantic path (m_n) is extremum if and only if for any other semantic path (m_n') $\forall n \in$, whenever m_n and m_n' cand be compared for the justification ordering , then $m_n \leq_{\star p}^{G(\cup_n \Phi_n)} m'n$. If $\star$ is a universal ionic operator, then any semantic path determines an element of $\Sigma(\Phi)$ and conversely [2].

The following variational principle (least action principle) is introduced: "*Of all the possible paths in* $\Sigma(\Phi)$ *that it might take to get from one state to another, the point representing a (physical) logic system takes a path* (m_n) *which is extremum, i.e., the smoothest possible on the characteristic surface representing the system*. This principle provides a solution to the frame problem.

Example: (Yale shooting problem). *After performing an action, things normally remain as they were. After loading a gun, the gun is loaded. After a man is shot with a loaded gun, he dies. After loading a gun, waiting and shooting Fred, is it the case that Fred will die?* The characteristic surface of this problem has 10 semantic paths, one of which is minimal among the other paths. When the evolution of the system follows tis path, Fred is dead after being shot.

Example: (Robot problem). *After performing an action, things normally remain the same. After locking the robot's forward gear, the gear is locked if it can be locked. After trying to move, the robot will move forward if the reverse gear is locked and the forward gear is unlocked.*

Aftyer trying to move, the robot will move backward if the forward gear is locked and the reverse gear is unlocked. If the robot is moving backward, then the reverse gear is unlocked. If the robot is moving forward, then the forward gear is unlocked. Initially the reverse gear is locked, the forward gear is unlocked, the robot is not moving at all, and the forward gear can be locked. After waiting, locking the forward gear, and trying to move the robot, is it the case that the robot stand still? The characteristic surface of this problem has one extremum path. When the evolution of the system follows this path, locking the forward gear was successful, and the robot is not moving forward and is not moving backward.

[1] M. A. NAIT ABDALLAH, *Commonsense reasoning = logic + partial information (abstract)*, ***Proceedings of the 1992 European Summer Meeting of the ASL***, 1992.

[2] ———, ***The logic of partial information***, EATCS Research Monographs in Theoretical Computer Science, Springer Verlag, 1994.

► ZOFIA ADAMOVICZ, *Consistency of* $\neg exp$ *and* $\exists^*_m$ *definability of all small* Σ_1 *definable numbers*.
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Let $\exists^*_m$ denote the collection of Σ_1 formulas of restricted complexity – these are translations of $\exists\Pi^b_m$ (where Π^b_m is Buss's class) formulas to a language without terms.

We prove that for a sufficiently large m there is a model of the theory $I\Delta_0 + \Omega_2 + \neg exp$ in which for every Σ_1 definition $\phi(x)$ there is an $\exists^*_m$ definition $\varphi(x)$ such that

$$\phi(x) \Leftrightarrow \varphi(x)$$

for sufficiently small x.

► DAVID ALBRECHT, FRANK A. BÄUERLE, JOHN N. CROSSLEY AND JOHN JEAVONS, *Curry-Howard terms for linear logic*.
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In [4] J. Y. Girard presented a Gentzen-style system for linear logic together with a strong normalization theorem based on proof-nets. In [3] Crossley and Shepherdson presented an extension of the Curry–Howard correspondence to full predicate logic (with possibly non-logical axioms). The Curry–Howard notation permits mechanization of the normalization process.

Now we present a natural deduction system of introduction and elimination rules for first order full linear logic, Curry–Howard terms for deductions in this logic and a strong normalization theorem which uses a modified version of Girard's candidates for reducibility.

The logical system is a further development of those of Troelstra [5] and Benton *et al.* [2] and the rules follow fairly traditional patterns.

On the other hand the new Curry–Howard terms involve substantial modifications to the lambda calculus, in particular the introduction of new operators ι, κ, τ and ω with their associated reduction rules. (The system of terms goes beyond that of Abramsky [1].) More importantly the definition of substitution (equivalent to replacing one sub–proof by another) has to be updated.

THEOREM 1. *There is a one-one correspondence between Curry–Howard terms and proofs in first order full linear logic.*

THEOREM 2. *Every Curry–Howard term for first order full linear logic strongly normalizes.*

[1] S. ABRAMSKY, *Computational interpretations of linear logic*, Technical Report 90/20, Department of Computing, Imperial College, London, October 1990.

[2] N. BENTON, G. BIERMAN, V. DE PAIVA, and M. HYLAND, *A term calculus for intuitionistic linear logic*, ***Typed lambda calculi and applications. proceedings of the international conference TLCA '93 March 16–18, Utrecht, The Netherlands***, LNCS, no. 664, Springer-Verlag, 1993, pp. 75–90.

[3] J. N. CROSSLEY and J. C. SHEPHERDSON, ***Extracting programs from proofs by an extension of the Curry-Howard process***, Birkhäuser, 1993.

[4] J.-Y. GIRARD, *Linear logic*, ***Theoretical Computer Science***, vol. 50 (1987), pp. 1–102.

[5] A. S. TROELSTRA, ***Lectures on linear logic***, no. 29, Stanford, CA: Center for the study of language and information, 1992.

► KLAUS AMBOS-SPIES, SEBASTIAAN A. TERWIJN AND XIZHONG ZHENG, *Resource bounded randomness and weakly complete problems*.

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We introduce and study resource bounded random sets based on Lutz's concept of resource bounded measure ([4,5]). We concentrate on n^c-randomness ($c \geq 2$) which corresponds to the polynomial time bounded (p-) measure and which is adequate for studying the internal and quantitative structure of $\mathbf{E}$=DTIME(2^{lin}).

First we show that the class of n^c-random sets has p-measure 1. This provides a new, simplified approach to p-measure 1-results.

Next we compare randomness with genericity (in the sense of [1, 2]) and we show that n^{c+1} - random sets are n^c-generic whereas the converse fails. From the former we conclude that n^c-random sets are not p-btt-complete for **E**.

Our technical main results describe the distribution of the n^c-random sets under p-m-reducibility. We show that every n^c-random set in **E** has n^k-random predecessors in **E** for any k$\geq$1 whereas the amount of randomness of the successors is bounded. We apply this result to answer a question raised by Lutz [3]: We show that the class of weakly complete sets has measure 1 in **E** and that there are weakly complete problems which are not p-btt-complete for **E**.

[1] K. AMBOS-SPIES, H. FLEISCHHACK, and H. HUWIG, *Diagonalizations over polynomial time computable sets*, ***Theoretical Computer Science***, vol. 51 (1987), pp. 177–204.

[2] ———, *Diagonalizing over deterministic polynomial time*, ***Proceedings of CSL 1987***, Lecture Notes in Computer Science, no. 329, Springer Verlag, 1988, pp. 1–16.

[3] J. H. LUTZ, *Weakly hard problems*, (preliminary version), April 1993, submitted for publication; extended abstract in ***Proceedings of the 9th Structure in Complexity Theory Conference***, IEEE Computer Science Press, 1994. pp. 146–161.

[4] ———, *Category and measure in complexity classes*, ***SIAM Journal of Computing***, vol. 19 (1990), pp. 1100–1131.

[5] ———, *Almost everywhere high nonuniform complexity*, ***Journal of Computer and System Science***, vol. 44 (1992), pp. 220–258.

► MATTHIAS BAAZ, CHRISTIAN G. FERMÜLLER AND RICHARD ZACH, *Proof theory of finite-valued logics*.

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Proof theoretic calculi for finite-valued logics (sequent calculi, tableaux, natural deduction) have been proposed several times since the 50s, but results about them have usually not gone further than soundness and completeness theorems.

The approach used is based on many-sided sequents, say, $\Gamma = \Gamma_1 \mid \dots \mid \Gamma_m$, if the logic under consideration is m-valued. Here, Γ_i is a finite sequence of formulas. Such a sequent is true in a given structure, if some Γ_i contains a formula A which takes the truth value i (in the structure). Based on this generalized notion of Gentzen's sequents, a calculus can be constructed for any finite-valued first-order logic with usual connectives and so-called distribution quantifiers [3]. The ordinary universal and existential quantifiers belong to this very general class of quantifiers. The construction is effective: for any finite-valued matrix a calculus can be computed (see [1]).

If Γ and Δ are two sequents, the Γ, Δ denotes the sequent obtained by concatenating the corresponding Γ_i and Δ_i. [i: A] is the sequent which is empty everywhere, except that it has the formula A at position i.

The calculus contains the following cut rule:

$$\frac{\Gamma, [i : A] \quad \Gamma', [j : A]}{\Gamma, \Gamma'}$$

where i and j are distinct. We show how many-valued resolution [2] can be used to obtain the cut-elimination theorem for such calculi, in a similar uniform way as the construction of the calculi itself. The proof also (obviously) works for the classical case and points the way to obtaining resolution-based interactive cut-elimination procedures.

[1] M. Baaz and C. G. Fermueller, *Resolution for many-valued logics*, ***Proceedings LPAR 1992***, LNAI, no. 624, Springer, pp. 107–118.

[2] M. Baaz, C. G. Fermueller, A. Ovtrucki, and R. Zach, *MULTLOG: A system for axiomatizing many-valued logics*, ***Proceedings LPAR '93,***, LNAI, no. 698, Springer, Berlin, 1993, pp. 345–347.

[3] W. A Carnielli, *Systematization of finite many-valued logics through the method of tableaux*, ***Journal of Symbolic Logic,***, vol. 52 (1987), pp. 473–493.

► S. A. BADAEV, *Some problems on computable enumerations*.
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Let $\mathcal{A}$ denote a class of r.e. sets. A surjective mapping $\alpha : \omega \to \mathcal{A}$ is called enumeration of $\mathcal{A}$. If the set $\{\langle n, x\rangle : x \in \alpha n\}$ is r.e., then enumeration α is called computable. Enumeration α is said to be reducible to enumeration $\beta(\alpha \leq \beta)$, if $\beta = \alpha f$ for some recursive function f If for every enumeration β of $\mathcal{A}\beta \leq \alpha$ implies $\alpha \leq \beta$, then α is called minimal enumeration of $\mathcal{A}$.

Theorem 1. *Enumeration α of $\mathcal{A}$ is minimal iff for every r.e. set W if $\alpha(W) = \mathcal{A}$ then there exists r.e. equivalence η such that $\forall x \forall y \exists z \in W((\langle x, y\rangle \in \eta \to \alpha x = \alpha y) \& \langle x, z\rangle \in \eta)$*

This criteria published in [1] was used widely for solving some problems of the theory of enumerations [3]. In particular, it had been obtained natural classification of minimal enumeration and had been round an elementary proof of an analogy of speed-up theorems [2]. One of the recent results in this direction is the following.

Theorem 2. *Let computable class $\mathcal{A}$ contain any pair of embedded sets $A \subset B$ such that the set $N \setminus B$ can be separated from the set $\mathcal{A}$ by some r.e. set. Then $\mathcal{A}$ has infinitely many pairwise* *non-reducible computable enumerations.*

[1] S. A. BADAEV, *About one of Goncharov's problems*, ***Siberian Mathematics Journal***, vol. 32 (1991), no. 3, pp. 532–534.

[2] ———, *On minimal enumerations*, ***Siberian Advances in Mathematics***, vol. 2 (1992), no. 1, pp. 1–30.

[3] YU. L. ERSHOV, ***Theory of enumerations***, Nauka, Moscow, 1977.

► JOAN BAGARIA AND ROGER BOSCH, *Definable fragments of Martin's Axiom*.

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By a definable poset we mean a poset belonging to one of the following classes: the class of projective posets, the class of constructive (from a given real) poset or the class of ordinal-definable (with a real as a parameter) poset.

We study the axioms that we obtain by restricting Martin's Axiom to one of these of definable posets. We construct models for the axioms by iterating only definable posets and we show that many of the consequences of Martin's Axiom do not hold in them. As a corollary, we obtain a much stronger form of a theorem of H. Judah and S. Shelah in [1].

[1] H. JUDAH and S. SHELAH, *Souslin forcing*, this JOURNAL, vol. 53 (1988), no. 4, pp. 1188–1207.

► STEFANO BARATELLA AND RUGGERO FERRO, *Non standard regular finite set theory*.

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Our goal is to introduce a set theory negating the cantorian axiom of infinity, but, at the same time, still able to support modern mathematics.

Lying underneath there is a notion of infinity, different from the cantorian one. Indeed we will admit the existence of natural numbers of which we do not recall the entire process of construction. These numbers may be regarded as infinite. So we get another notion of infinity that takes into account the role of the observer.

In the theory **NRFST** (*Non standard Regular Finite Set Theory*) that we propose, in addition to the usual membership predicate, there is an undefined unary predicate, *standard*, whose intended interpretation is that it holds on the elements that can be grasped by the observer.

All the elements of the observed world form the *internal universe*.

We will assume that the observer has a vague notion of the non standard elements of the observed world. To be able to consider these vague notions we need a further undefined unary predicate, *internal*, that will distinguish between the objects of the observed world and the extensions of vague notions associated to them.

In our presentation we would like to introduce the axioms of **NRFST**, their heuristic motivations, and to report on the basic features and relative consistency results of theory.

► CATALIN BARBACIORU, *The topology of implicative BCK-algebras*.

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A BCK-algebras is an algebra of type $\langle 2, 0\rangle$ which satisfies the following axioms:

1) $((x^*y)^*(x^*z)^*(z^*y)) = 0$

2) $(x^*(x^*y)^*y) = 0$

3) $x^*x = 0$
4) $0^*x = 0$
5) $x^*y = y^*x = 0$ imply $x = y$.

For an implicative BCK-algebra $(X,^*, 0)$ any prime ideal is maximal and is characterized by the property that for any x, y in I either x^*y or y^*x is in I. The set of prime ideals is denoted by $B(X)$.

The family $(S_X(\mathcal{X}))$ is a topological base of compact open sets of $B(X)$, where $S_X(\mathcal{X}) = (I \in B(X) : \mathcal{X} \notin I)$.

Denoting by s the topology defined by this base then the topological space $(B(X), s)$ verifies the following properties :

i) admits a topological base of compact open sets;

ii) for any compact open sets U, V of $B(X)$ the set $U \cap *(B(X) - V)$ is a compact open set;

iii) for any distinct I, J in $B(X)$, there exists a compact open set U such that $I \in U$ and $J \notin U$.

Let $f : X \to Y$ be a morphism of implicative BCK-algebras. We denote by $B(f)$ the map $B(f) : B(Y) \to B(X)$ defined by $B(f)(J) = f^{-1}(J)$. Then:

a) if V is a compact open set of $B(X)$ then $B(f)^{-1}(V)$ is a compact open set of $B(Y)$ (so $B(f)$ is a continuous map);

b) for any compact set U of $B(X)$ we have $B(f)^{-1}(*(B(X) - U)) = *(B(f)^{-1}(B(X) - U))$.

Let C be the category whose objects are the topological spaces with properties i), ii), iii) and whose morphisms are the maps which satisfy a), b). Then B becomes a contravariant functor from the category of implicative BCK-algebras into the category C. For an object A of C let $T(A) = (U \subseteq A :$ is a compact open set).

We define $U^*V = U \cap (A - V)$ for any U, V in $T(A)$. Then $(T(A), *, 0)$ is an implicative BCK-algebra. For A, B in ObC and f in $Hom_c(A, B)$ we define $T(f) : T(B) \to T(A)$ by $T(f)(V) = f^{-1}(V)$. Then T becomes a contravariant functor. Furthermore the maps $(S_X)_{X \in BCK_i}$ define a functorial morphism S between the covariant functors 1_{BCK_i} and $T \circ B$.

We remark that the one-to-one morphism S_X is an isomorphism iff X is a superior semilattice.

We may conclude that the dual category BCK_i^v, of superior semilattices positive implicative BCK-algebras, is the subcategory of C, image of BCK_i^v, through the functor B.

► ANATOLIJ P. BELTIUKOV, *An intuitionistic formal theory with realizability in polynomial time*.
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A formal intuitionistic theory is proposed (equivalent to the Peano arithmetic by expressive power) with realizability of proved formulas in polynomial time. It can be expressed and proved in the theory, that the polynomially bounded Turing machines (with polynomial time counter) really complete their work and that the results of this work really exist. A POLYNOMIAL total algorithm f with the property

$$\forall x\ A(x, f(x))$$

can be algorithmically extracted from any proof of any formula

$$\forall x\ A(x, f(x))$$

in the theory. Here x and y can be vectors.

The theory is intended to use for deductive synthesis of algorithms with bounded computational complexity. The matter is that algorithms with the same property can be extracted from ordinary intuitionistic proofs but these algorithms may be so hardly computable that their usage in practical programming is impossible.

The theory operates with words over some finite alphabet, particularly, with numbers in the binary notation. It uses operations of concatenation and word multiplication

$$(w\widehat{\ }v = w \dots w \ (|v| \text{ times}), \text{ where } |v| \text{ is the length of } v),$$

word equality, "subword" relation, lexicographical word ordering. The theory has a sufficiently large axiom system to describe main properties of these operations and relations. The main difference between the theory and Peano arithmetic is induction axiom scheme restriction. Only the WORD INDUCTION of the following form is permitted:

$$A() \wedge \forall x \ (A(x) \longrightarrow A(xs_1) \wedge \dots \wedge A(xs_n)) \longrightarrow (x)A(x)$$

where $\{s_1, \dots, s_n\}$ = the alphabet with which words the theory works. All the quantifiers of formulas, by which induction can be fulfilled, must be bounded. All values of existentially quantified variables situated outside negated formulas and premises of implications must be less (in lexicographical order) than some terms in the language

$$(\exists x \ (x < t \wedge A(x))).$$

The values of other quantified variables must be bounded by subword relation with some terms

$$((Qx)(x \text{ subword } t \wedge A(x))).$$

These boundedly quantified formulas describe precisely NP.

► VLADIMIR Y. BELYAEV, *HF-logic, computability over reals and other structures*.
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I consider the formalization of algorithm notion over any model A of finite language L which use BSS-machines [3] and notion of the list superstructure $HL(A)$ [1]. BSS-machines include as base operations the computabilities of terms of language L and relations of L in branch nodes. It is not possible to use elements of A except input elements, constants from L and values of terms from its. $HL(A)$ is elements of $|A|$, finite lists of elements from $|A|$, finite lists of elements and lists and so on. The necessary operations are added to L. $HL(A)$ has obvious similarity with $HF(A)$ – the minimal admissible set with urelements from A. This analogy leads to natural interdependence of HF-logic and computability over A. In particular we have the characterization of relations on A definable by formulas of HF-logic in recursion theory terms.

Let $\Sigma_0^s = \Sigma_0^r$ – class of recursion relations on $HL(A)$, Σ_{n+1}^s (Σ_{n+1}^r) is class of domains (range) of BSS-computable functions with oracle from Σ_n^s (Σ_n^r). This hierarchies was defined by author and A. Myasnykov in [2].

THEOREM 1. *Let $X \subseteq |A|^k$. Then X is definable by Σ_n-formula of HF-logic iff X is Σ_n^r-relation.*

Let A be the reals ($L = \{+, \cdot, -, ^{-1}, 0, 1, \leq\}$).

THEOREM 2. *In A the classes Σ_n^s and Σ_n^r are hierarchies of Borel and projective sets respectively in **effective** descriptive set theory.*

[1] I. V. ASHAEV, V. Y. BELYAEV, and A. G. MYASNIKOV, *Approaches to the theory of generalized computability*, ***Algebra i Logika***, vol. 32 (1993), no. 4, pp. 349–386.

[2] V. BELYAEV and A. MYASNIKOV, *Theorems on hierarchy in generalized recursion theory*, to appear.

[3] L. BLUM, M. SHUB, and S. SMALE, *On a theory of computation and complexity over the real numbers: NP-completeness*, ***Bulletin of the American Mathematical Society***, vol. 21 (1989), no. 1, pp. 1–47.

► A. BERARDUCCI AND M. OTERO, *A recursive nonstandard model of normal open induction*.
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Models of (normal) open induction are those (resp. normal) ordered domains which are an integral part of the real closure of their fraction field.

In 1964 Shepherdson gave a recursive nostandard model of open induction. His model is not normal and does not have infinite prime elements.

Here we present a recursive nonstandard model of normal open induction with an unbounded set of infinite prime elements. Answering questions posed by Macintyre and Marker in [2]and Kaye in [1]. The construction is done using techniques of Shepherdson in [3] and Wilkie in [4].

[1] R. KAYE, *Open induction, Tennenbaum phenomena, and complexity theory*, ***Arithmetic, proof theory and computational complexity*** (Clote and Krajicek, editors), Oxford Logic Guides, no. 23, Oxford, 1993, pp. 222–237.

[2] A. MACINTYRE and D. MARKER, *Primes and their residue rings*, ***Annals of Pure and Applied Logic***, vol. 43 (1989), pp. 57–77.

[3] J. SHEPHERDSON, *A nonstandard model for a free variable fragment of number theory*, ***Bulletin de L'Academie Polonaise Science***, vol. 12 (1964), pp. 79–86.

[4] A. WILKIE, *Some results and problems on weak systems of arithmetic*, ***Logic colloquium '77*** (Mcintyre et al., editors), North-Holland, 1978, pp. 285–296.

► G. M. BIERMAN AND VALERIA C. V. DE PAIVA, *Yet another intuitionistic modal logic*.
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In this work we investigate the categorical proof theory of a intuitionistic version of the modal logic **S4**. The formulation of intuitionistic versions of well-known classical modal logics is far from settled in the literature. The most popular approach is to adapt your favourite intuitionistic model to suggest the theorems which the modal logic should satisfy. The problem with this approach is that different adaptations and different choices of models results in different, *non-equivalent*, intuitionistic modal logics.

In this work we propose an intuitionistic modal logic, $\mathbf{S4}_p$, for which we provide equivalent axiomatic, sequent calculus and natural deduction formulations. It turns out that our proposal is equivalent to that proposed by Prawitz in his monograph (hence the 'p' subscript). However our natural deduction formulation is significantly more elegant than that proposed by Prawitz.

Another contribution of this work is the development of a notion of a categorical model of $\mathbf{S4}_p$. It turns out that some interesting variants of well-known categorical concepts need to be used. It should be possible to use these new categorical concepts, à la Kripke-Joyal, to suggest new Kripke-like structures.

► TERRY BOSWELL, *Historical reflections on the logical analysis of quasi-definite descriptions*.
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It is characteristic of Frege's logic that such phrases as "some animal", which are sometimes termed quasi-definite descriptions, are analyzed in the literal sense of the word. They resolve and disappear upon analysis. This is one aspect of his rejection of the subject-predicate analysis of statements, and constitutes one of his major achievements in logic. Whereas in Frege's analysis of such phrases no logical structure remains which corresponds directly to them, logicians since his time have often attempted to reconstruct forms within a Fregean framework which do so. This has partly been done in the interest of making logic more "natural", as for example when a natural deduction system allows the choice of single letters which stand for such phrases. But it is also for example characteristic of Hilbert's ε-calculus and of the generalized theory of quantifiers. Certain pre-Fregean uses of symbolism in logic, for example in Aristotle and Leibniz, have also retained such a correspondence.

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With each of the usual tree forcings $\mathcal{I}$ (e.g., $\mathcal{I}$ = Sacks forcing $\mathcal{S}$, Laver forcing $\mathcal{L}$, Miller forcing $\mathcal{M}$, Mathias forcing $\mathcal{R}$, etc.) we associate a σ-ideal i^0 on the reals as follows: $A \in i^0$ iff for all $T \in \mathcal{I}$ there is $S \leq T$ (i.e. S is stronger than T or, equivalently, S is a subtree of T) so that $A \cap [S] = \emptyset$, where $[S]$ denotes the set of branches through S. (so, s^0 is the ideal of Marczewski null sets, r^0 is the ideal of Ramsey null sets (nowhere Ramsey sets) etc.)

We show (in ZFC) that whenever i^0, j^0 are two such ideals, then $i^0 \setminus j^0 \neq \emptyset$. E.g., for $\mathcal{I} = \mathcal{S}$ and $\mathcal{J} = \mathcal{R}$ this gives a Marczewski null set which is not Ramsey, extending earlier partial results by Aniszczyk, Frankiewicz, Plewik, Brown and Corazza and answering a question of the latter.

For these ideals i^0, we will also discuss the covering number $\mathrm{cov}(i^0)$, the smallest size of a family of sets from i^0 covering the reals.

► DUMITRU BUSNEAG, *Bounded Hilbert and Hertz algebras of fractions*.
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A *Hilbert algebra* (Ha) is a triple $(A, \to, 1)$, where A is a non-empty set, $\to$ is a binary operation on A, $1 \in A$ is an element such that the next axioms is verified for every $x, y, z \in A$:

(1) $x \to (y \to x) = 1$,

(2) $(x \to (y \to z)) \to ((x \to y) \to (x \to z)) = 1$,

(3) If $x \to y = y \to x = 1$ then $x = y$.

If A is a Ha , then the relation $x \leq y$ iff $x \to y = 1$ is a partial order on A (called *natural ordering* on A); with respect to this ordering, 1 is the largest element of A. A *bounded Ha* (bHa) is a *Ha* with a smallest element 0 .

A *Hertz algebra* (Hza) is a Ha H with the property that for every $x, y \in H$ the infimum $x \wedge y$ (relative to the natural ordering) exists in H.

Examples:

(1) If $(B, \vee, \wedge, \neg, 0, 1)$ is a Boolean lattice, then $(B, \to, \wedge, 0, 1)$ is a bounded Hertz algebra, where $\to$ is defined by $x \to y = (\neg x) \vee y$, for $x, y \in B$.

(2) If $(X, \mathcal{T})$ in a topological space, then $(\mathcal{T}, \to, \cap, X, 0)$ is a bounded Hertz algebra, where for $D, D' \in \mathcal{T}$, $D \to D' = \mathrm{int}\,((X - D) \cup D')$, $1 = X$ and $0 = \emptyset$.

The origin of Hilbert algebras is in mathematical logic. In 1923, Hilbert was the first who remarked the possibility of study a very interesting part of the classical propositional calculus, taking as axioms only the Hilbert's ones concerning only for logical implication (this field is known as "*positive implicative propositional calculus*").

In the rest of this resume, by A we denote a bHa. For $x, y, \in A$, we denote by

$x \mid\sqcup y = x^* \to y$, where $x^* = x \to 0$ (we have $x, y, \leq x \mid\sqcup y$).

A Ha A' is called *Hilbert algebra of fractions of A* if :

(1) A is a Hilbert subalgebra of A' and

(2) For every $a', b', c' \in A', a' \neq b'$, there exists $a \in A$ such that $a \mid\sqcup a' \neq a \mid\sqcup b'$ and $a \mid\sqcup c' \in A$ (as a notational convenience, we write $A \leq A'$ to indicate that A' is a Ha of fractions for A). $Q(A)$ is the *maximal Ha of quotients of A* if $A \leq Q(A)$ and for every A' with $A \leq A'$, then there exists an injective morphism of Hilbert algebras i : $A' \to Q(A)$.The notions of *Hertz algebra of fractions* and *maximal Hertz algebra of quotients* is defined in an analogous way as for Hilbert algebras. In [D. Busneag : Hilbert algebra of fractions and maximal Hilbert algebras of quotients, Kobe Journal of Math., 5(1988), pp. 161–172] and [D. Busneag : Hertz algebras of fractions and maximal Hertz algebra of quotients, Mathematica Japonica, 3(1993), pp. 1–8] we have proved the existence of $Q(A)$ and $Q(H)$ (H a bHza).

The aim of this communication is to prove the next result: *For every bHa A (resp. bHza H) there exists a non-empty set I (resp. I') and a filter F (resp. F') on I (resp. I') such that $Q(A)$ (resp. $Q(H)$) is a subalgebra of a reduced power A^I/F (resp. $H^{I'}/F'$).*

► ALEXANDER CHAGROV, *The undecidability of the tabularity problem for modal logics*.
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A modal logic is called *tabular* if it is characterized by a finite model (or frame). Let $L + \varphi$ ($L \oplus \varphi$) be the closure of $L \cup \{\varphi\}$ under substitution and modus ponens (respectively, substitution, modus ponens and necessitation) and K_n the minimal normal modal logic with n necessity operators $\Box_1, \dots, \Box_n$.

THEOREM 1. (i) *For no $n \geq 1$, there is an algorithm which can determine, given a formula φ, whether $K_n + \varphi$ is tabular.*

(ii) *For no $n \geq 1$, there is an algorithm which can determine, given a formula φ, whether $K_n \oplus \varphi$ is tabular.*

THEOREM 2. (i) *Let L be a tabular extension of K_n, for $n \geq 1$. There is no algorithm which can determine, given a formula φ, whether $K_n + \varphi = L$. In particular, the consistency problem for extensions of K_n is undecidable.*

(ii) *Let L be a normal tabular extension of K_n, for $n \geq 2$. There is no algorithm which can determine, given a formula φ, whether $K_n \oplus \varphi = L$. In particular, the consistency problem for normal extensions of K_n is undecidable.*

THEOREM 3. *Suppose L is a normal tabular extension of K_1 such that $L \not\vdash \Diamond\top$. There is no algorithm which can recognize, given a formula φ, whether $K_1 \oplus \varphi = L$.*

It is well known (D. Makinson) that the consistency problem for normal extensions of K_1 is decidable. The second (and the last) decidable property of K_1's extensions known to me is the property of coincidence with K_1.

It may be of interest to note that in parallel with Theorem 3 one can prove

THEOREM 4. *Let L be a consistent normal finitely axiomatizable extension of the provability logic G (e.g., G, G3, $G \oplus \Box\bot$). There is no algorithm which can recognize, given a formula φ, whether $K_1 \oplus \varphi = L$.*

► LILIA A. CHAGROVA, *Undecidability of the first-order definability of intuitionistic formulas on finite frames*.
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It is well-known that there are difficulties when we transfer negative results about modal formulas to intuitionistic ones. For examples, remind Shehtman's result: there are normal extensions of $S4Grz$, which are incomplete with respect to the topological semantics, but analogous examples for superintuitionistic logics aren't known.

A purpose of the present report is to transfer the main result from [1] to intuitionistic case.

THEOREM 1. *The problem of recognizing, given any intuitionistic formula, whether it is first-order definable on the class of all finite intuitionistic frames, is algorithmically undecidable.*

[1] A. V. CHAGROV and L. A. CHAGROVA, *Algorithmic problems concerning first-order definability of modal formulas on the class of all finite frames*, Preprint ML-93-07. University of Amsterdam, 1993.

► WITOLD CHARATONIK, *Solving set constraints with projections*.
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Set constraints give a natural formalism for many problems in program analysis and type inference. The first general results concerning the decidability of set constraints were obtained by Heintze and Jaffar , who studied the so-called definite set constraints. The decidability of systems of set constraints without projections was established by A. Aiken and E.L. Wimmers. Later other proofs have been obtained. R. Gilleron, S. Tison, and M. Tommasi gave a proof based on automata theoretic techniques and L. Bachmair, H. Ganzinger, and U. Waldmann [1] gave a simple and very elegant proof using the decision procedure for the first order theory of monadic predicates. One more algorithm was presented by A. Aiken, D. Kozen, M. Vardi, and E.L. Wimmers.

The problem of decidability of systems of set constraints with projections is much more difficult. There were three different approaches to solve this problem. In each of them the first step toward a general solution was solving the problem of decidability of negative set constraints – a special case of set constraints with projections. Three independent proofs were given by Aiken, Kozen and Wimmers, Gilleron, Tison and Tommasi, and Charatonik and Pacholski [2]. So far only our approach gave the solution of the general problem.

Our method is an extension of the results presented in [1] and [2]. In [1] the case of negative occurrences of projection symbols was solved by introducing for each projection expression a predicate symbol encoding this expression. In case of unrestricted occurrences of projection symbols the same predicate gives only some approximation of the desired set. Extending the methods of [2] we are able to check if this approximation is correct. The first observation is that each solution of a system of set constraints gives a finite set called a *history* which is sufficient to reconstruct a (perhaps different) regular solution. Then, the proof proceeds by distinguishing a small subset, *the skeleton* of a history, which is small, is not always a history, but can be collapsed to obtain a small history. Now, what is left, is to make an exhaustive search through all *small histories*.

The results appeared in W. Charatonik and L. Pacholski, *Set constraints with projections are in NEXPTIME*, 35th annual IEEE symposium on foundations of computer science, 1994, pp. 642–653.

[1] L. BACHMAIR, H. GANZINGER, and U. WALDMANN, *Set constraints are the monadic class*, ***Eighth annual IEEE Symposium on Logic in Computer Science***, 1993, pp. 75–83.

[2] W. CHARATONIK and L. PACHOLSKI, *Negative set constraints with equality*, ***Ninth annual IEEE symposium on logic in computer science***, 1994, pp. 128–136.

► MARCELLO D'AGOSTINO AND MARCO MONDADORI, *Classical logic as the limit of a sequence of cut-bounded feasible logics*.
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In this paper we address the problem of coping with the exponential worst-case complexity of proof-search in analytic systems of deduction developed in the Gentzen tradition. Elsewhere [*The taming of the cut*, in Journal of Logic and Computation vil. 4, 1994, pp. 285–319] we have proposed a new notion of classical deduction based on a system called KE and proved that even its *analytic* restriction is essentially more efficient than the tableaux method. KE is a refutation system, like the tableaux method, but all its "operational" rules are linear while the *only* branching rule is a "structural" rule expressing a form of cut (which we call PB). A natural strategy to cope with the problem of exponential worst-case complexity consists in imposing an upper bound on the number of applications of this rule. Call $KE(n)$ the system obtained from KE by allowing at most n analytic applications of PB. We show that :

• $KE(1)$ has a decision procedure which runs in time $O(n^2)$.

• For every fixed n, $KE(n)$ has a polynomial time decision procedure.

Obviously, the set of classical tautologies is obtained as the limit of the sets of tautologies of each $KE(n)$. The interesting point is that low-depth systems are powerful enough for a wide range of applications. In the final part of the paper, we investigate alternative characterizations of the sets of tautologies of each $KE(n)$.

► G. D'AGOSTINO, A. MONTANARI AND A. POLICRITI, *Translating modal formulae as set-theoretic terms*.
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Translation methods for modal logics into classical first-order logic allow to use mechanical theorem provers for predicate calculus as modal theorem provers. A common feature of all the existing translation methods is that in order to be applied directly, the underlying modal logic must have a first-order semantics [2]. The main motivation of the present work is to find a translation method applicable to all normal complete finitely-axiomatizable modal logics, regardless of whether they are first-order complete or not, and whether a semantic characterization is given or not for them. In [1] we propose a set-theoretic translation method that represents any Kripke frame as a set, with the accessibility relation modeled using the membership relation $\in$. The whole translation is carried out in the following finite set theory Ω:

$$x \in y \cup z \leftrightarrow x \in y \vee x \in z, \qquad x \in y \setminus z \leftrightarrow x \in y \wedge x \notin z,$$
$$x \subseteq y \leftrightarrow \forall z(z \in x \to z \in y), \qquad x \in Pow(y) \leftrightarrow x \subseteq y.$$

Ω does not contain the extensionality axioms as well as the axiom of foundation. We will make an essential use of the latter fact since we will be forced to work in universes containing non-well-founded sets and it will be convenient to use universes satisfying AFA.

A modal formula $\phi(P_1, \ldots, P_n)$ is mapped in a set-theoretic term $\phi^*(x, x_1, \ldots, x_n)$ inductively defined as follows:

$$P_i^* \equiv x_i, \qquad (\phi \vee \psi)^* \equiv \phi^* \cup \psi^*, \qquad (\neg\phi)^* \equiv x \setminus \phi^*, \qquad (\Box\phi)^* \equiv Pow(\phi^*).$$

We prove that for $H = K + \psi(P_{j_1}, \ldots, P_{j_m})$, where $\psi(P_{j1}, \ldots, P_{jm})$ is an axioms' schema and K is the minimal normal modal logic, the following holds:

$\vdash_H \phi(P_1, \ldots, P_n)$ iff

$$\Omega \vdash \forall x(Trans(x) \wedge Axiom_H(x) \rightarrow \forall x_1 \ldots \forall x_n(x \subseteq \phi^*(x, x_1, \ldots x_n))),$$

where $Trans(x)$ is the formula $\forall y(y \in x \rightarrow y \subseteq x)$ and $Axiom_H(x)$ is the formula

$$\forall x_{j_1}, \ldots, \forall x_{j_m}(x \subseteq \psi^*(x, x_{j1}, \ldots, x_{jm})).$$

The finite axiomatizability of Ω guarantees the possibility of implementing mechanical proof search procedures via the deduction theorem or more specialized techniques. In particular, we are currently exploring the possibility of applying theory resolution techniques incorporating decidability results we obtained about Ω [1]. Moreover, the translation into set-theoretic terms, built using $U, \setminus, Pow$, guarantees that the translation of any formula is carried out in the same set-theoretic class of formulae, which does not contain equality (as well as Ω). The set-theoretic class of formulae corresponding to a given modal logic H is therefore completely defined by the translation of the axioms of H. The above points mark the difference between our method and more naive techniques that could be proposed to map modal formulae in a set-theoretic setting, passing (for example) through second order logic.

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In this paper we study 0-1 laws for extensions of first-order logic by Lindström quantifiers. We state necessary and sufficient conditions on a quantifier Q expressing a graph property, for the logic FO[Q] – the extension of first-order logic by means of the quantifier Q – to have a 0-1 law.

We use these conditions to show, in particular, that

FO[Ham], where Ham is the quantifier expressing Hamiltonicity, does not have a 0-1 law, and that FO[Rig], where Rig is the quantifier expressing rigidity, does have a 0-1 law. Blass and Harary [1] pose the question whether there is a logic which is powerful enough to express Hamiltonicity or rigidity and which has a 0-1 law (see also [2]). It is a consequence of our results that there is no such regular logic (in the sense of Ebbinghaus [3]) in the case of Hamiltonicity, but there is one in the case of rigidity. This is because, FO[Q] is the minimal regular logic that can define the property expressed by Q.

We also consider sequences of vectorized quantifiers, and show that the extensions of first-order logic obtained by adding such sequences generated by the quantifiers for planarity and 3-colourability both have 0-1 laws.

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► JAMSHID DERAKHSHAN, *The structure of topologically CZ-groups satisfying local periodicity conditions*.

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Recall that a group is called (topologically) CZ (as a generalization of linear groups) if it carries a Noetherian T_1-topology such that $x \longrightarrow x^{-1}$, $x \longrightarrow ax$, xa, and $x \longrightarrow a^x$ (for any a) are continuous. Since their introduction by Kaplansky in 1951 [5], a central question has been to determine their structure under additional conditions and to discover the discrepancies between their structure and that of (linear) algebraic groups. Note that there are non-algebraic groups which are CZ in the verbal topology (see for example [1]).

We present a solution of the above problem when the groups satisfy local finiteness conditions. Our results do not use the classification of finite simple groups. We first give a direct proof of the nilpotency of connected locally finite CZ-groups of bounded exponent. This generalizes the results of Bryant-Wilson [2], Bryant-Yen [3], Higgins [4] and Wehrfritz [7]. Otto Kegel [6] provides a proof in the case of the chain condition on centralizers using the classification of finite simple groups. It turns out that under certain conditions the simple CZ-groups are just $PSL_2(K)$ for K an infinite locally finite field of odd characteristic. The linearity follows from much fewer assumptions. We prove that $\aleph_0$-categorical CZ-groups with *max–cc* and definable multiplication are abelian by finite. We conjecture that groups of finite Moley rank (or stable) are CZ in some (definable) CZ-topology.

The Burnside conjecture that periodic groups are locally finite fails for CZ-groups, although they are quasi-compact. An application of the Zelmanov Lie algebraic theory of compact groups [8] to study the structure of periodic stable CZ-groups and periodic CZ-skew linear groups appears to be a promising direction for further research.

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A partition relation of pairs of reals is used to obtain a consistency result about perfect sets of real numbers. Sacks forcing and arguments of A. Miller are used to show the consistency of the partition relation of pairs, and another forcing argument is used to prove the following:

Theorem 1. *Con(ZFC + There is an inaccessible cardinal) implies Con(ZF + DC + There is an ultrafilter on ω + Every set of real numbers contains or is disjoint from a perfect set).*

This theorem also follows from a result of J. Henle which makes use of the dual Ramsey property of Carlson and Simpson.

We will comment on ways in which the theorem can be extended.

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► V. P. DOBRITSA, *Efficiency and set of computable indexations*.
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Introducing of an efficiency into abstract structures such as groups, algebras and models is one of the basic subjects of the theory of constructive models. Various and deep results had been obtained in this direction by efforts of many authors from different countries. Consideration of classes of constructive models with respect to their uniform efficiency leads to the notion of computable of constructive models. Now the theory of computable classes of constructive models received a large development effort.

The following stage of such kind of considerations is to study the set of computable indexations of computable class of constructive models. This set under the direct sum of computable indexations forms upper semilattice. Connections between efficiency and the set of computable indexations are given in the following statement.

Theorem. a) *There exists computable class of constructive models with non-computable set of computable indexations.*

b) *There exists computable class constructive models with computable set of computable indexations, but whose upper semilattice of computable indexations is not constructivizable.*

c) *There exists computable class of constructive models, whose upper semilattice of computable indexations is constructivizable.*

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Let us define the *intuitionistic part* of a classical theory T as the intuitionistic theory whose proper axioms are identical to the proper axioms of T. For example, Heyting arithmetic HA is the intuitionistic part of classical Peano arithmetic PA.

It's a well-known fact, proved by Heyting and Myhill, that the excluded middle can be derived in the intuitionistic part of ZF. So ZF is equal to its intuitionistic part.

In this paper, we mainly prove that TT, Russell's Simple Theory of Types, and NF, Quine's "New Foundations", are not equal to their intuitionistic part. So, an intuitionistic version of TT or NF, is more naturally obtained than an intuitionistic version of ZF.

In the first section, we present a simple technique to build Kripke models of the intuitionistic part of TT. This technique is a simplification of Fitting's forcing-like method (see [1]). We illustrate it with short examples bringing to light bad properties of finite sets if we keep Russell's definition (i.e., the set of finite sets is the smallest set F such that $\emptyset \in F$ and $x \in \{z\} \in F$ for all $x \in F$ and for all z).

The remaining sections are devoted to NF and one of its simple consistent fragment, NF_2. For each of these theories, we exhibit two suitable classical models, and show how to combine them in order to obtain non trivial Kripke models of the intuitionistic restrictions of NF_2 and NF. More precisely, in the case of NF, we use a result of Körner ([2]) to obtain two models $\mathcal{M}$ ond $\mathcal{M}'$ of classical NF, such that, on the one hand, $\mathcal{M} \models \varphi[a_1, \dots, a_n]$ iff $\mathcal{M} \models \varphi[a_1, \dots, a_n]$, for all *stratified* formulæ φ and all $a_1, \dots, a_n \in M$, and, on other hand, $\mathcal{M} \in \sigma$ and $\mathcal{M}' \not\models \sigma$ for some well-chosen *non-stratified* sentence σ. Then by putting $\mathcal{M}$ "on top of" $\mathcal{M}$, we obtain a Kripke model of the intuitionistic part of NF, where the excluded middle is not satisfied for some non-stratified sentence.

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► RYSZARD FRANKIEWICZ AND PAWEL ZBIERSKI, *Fat P-sets in the space ω^**.
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We prove that—consistently—in the space $\omega^* = \beta\omega \setminus \omega$ there are no P-sets with the $\mathfrak{c}$-*cc* and any two fat P-sets with the $\mathfrak{c}^+$-*cc* are coabsolute. Moreover, each fat P-set with the $\mathfrak{c}^+$-*cc* has a π-base tree of height ω.

► MARCELO F. FRIAS, ARMANDO M. HAEBERER, PAULO A. S. VELOSO AND GABRIEL A. BAUM, *Representability of fork algebras*.
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We consider *proper relation algebras* (PRA)

$$\mathcal{P} = \langle P, \cup, \cap, |, \sim, \prime, \emptyset, \Delta, V\rangle$$

of binary relations over a set $\mathcal{U}$, with *composition* $|$, *transposition* $\sim$, *diagonal relation* Δ and *universal relation* V as usual. Their abstract versions are *relational algebras* (RA's)

$$\mathcal{R} = \langle R, +, \bullet, ;, -, \smile, 0, 1, \infty\rangle$$

characterized by the usual set of axioms [2].

Consider a set $\mathcal{U}$ with an injective operation $\star : \mathcal{U} \times \mathcal{U} \to \mathcal{U}$ (constructing pairs $y \star z$). This induces an operation on subsets of $\mathcal{U} \times \mathcal{U}$:

$$r \lhd s = \{\langle x, y \star z\rangle \in \mathcal{U} \times \mathcal{U} : \langle x, y\rangle \in r \wedge \langle x, z\rangle \in s \wedge \langle y, z\rangle \in V\}.$$

A *proper fork algebra* (PFA) is a structure

$$\mathcal{F} = \langle F, \cup, \cap, |, \lhd, \sim, \prime, \emptyset, \Delta, V\rangle$$

such that F is closed under $\lhd$ and the reduct

$$\mathcal{F} = \langle F, \cup, \cap, |, \sim, \prime, \emptyset, \Delta, V\rangle$$

is a PRA.

The abstract version of a *PFA* is an *abstract fork algebra* (AFA): a structure

$$\mathcal{A} = \langle A, +, \bullet, ;, \nabla, -, \smile, 0, 1, \infty\rangle$$

such that

$$\langle A, +, \bullet, ;, -, \smile, 0, 1, \infty\rangle$$

is an RA and the following axioms hold:

(1) $r\nabla s = (r;\pi) \bullet (s;\rho)$,
(2) $(r\nabla s);(p\nabla q)^\smile = (r;p^\smile) \bullet (s;q^\smile)$,
(3) $\pi\nabla\rho \preceq 1$, and
(4) $0 \neq t \preceq \infty \otimes \infty \to (\exists v)(\exists w)(0 \neq v \otimes w \preceq t)$;

where *projections* and *cross* product are defined by

$$\pi = (1\nabla\infty)^{\smile} \quad \rho = (\infty\nabla 1)^{\smile} \text{ and } r \otimes s = (\pi; r)\nabla(\rho; s).$$

These projections satisfy the properties of quasi-projections [3], but are rather special. In fact, every atom of an atomic *AFA* is functional.

Other properties of atomic *AFA*'s are: $r \neq 0 \to \pi; r \neq 0 \wedge \rho; r \neq 0$, $r \otimes s = 0 \leftrightarrow r;\infty;s = 0$, $r \otimes s \neq 0 \to \breve{\pi};(r \otimes s);\pi = r \wedge \breve{\rho};(r \otimes s);\rho = s$, $(r \otimes s = t \otimes q) \wedge r \neq 0 \wedge s \neq 0 \wedge t \neq 0 \wedge q \neq 0 \to (r = t \wedge s = q)$,

$r \otimes s$ is an atom iff both r and s are atoms with $r;\infty;s \neq 0$, and if r is an atom with $r \preceq \infty\nabla\infty$ then $r = (r;\pi)\nabla(r;\rho)$.

THEOREM 1 (Representability). *Every (complete) atomic* AFA *is isomorphic to a* PFA [1]. (Thus we have a finitely axiomatizable algebra for Classical First-Order Logic with Equality [4]).

An isomorphism $\mathcal{H}$ is defined as in [2] based on certain pairs of atoms of 1, namely those $\langle\alpha, \beta\rangle$ with $\alpha;\infty;\beta \neq 0$. On them, $\otimes$ is an injective operation, as required. The crucial steps now are $\mathcal{H}(\infty) = V$ and $\mathcal{H}(r\nabla s) = \mathcal{H}(r) \lhd \mathcal{H}(s)$.

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We present a modification of the existing constructions of core models for large cardinals which allows for a smoother fine structure theory.

Our hierarchy is based on embeddings rather than extenders and is designed to allow truncation and condensation properties close to those present in the L-hierarchy.

It is hoped that with such a hierarchy the the coding method for L can be lifted without difficulty to core models.

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Let $\mathcal{L} = \{\neg, \to, \wedge, \vee\}$, be a propositional language of type (1,2,2,2) and let $A_3 = (\{0, 1/2, 1\}, \neg, \to, \wedge, \vee)$ be the three-element Lukasiewicz algebra. The three-valued Lukasiewicz deductive system is the pair $S_3 = (\mathcal{L}, \vdash_3)$, where the consequence relation $\vdash_3$ is defined in the following way: for all $\Sigma \cup \{\varphi\} \subseteq \mathcal{F}m_{\mathcal{L}}$, $\Sigma \vdash_3 \varphi$ iff for all $h \in hom(\mathcal{F}m_{\mathcal{L}}, A_3)$, $h(\Sigma) \subseteq \{1\}$ implies $h\{\varphi\} = 1$.

By using [5] we obtain an axiom system for a three-dimensional Gentzen system $\mathcal{G}_3 = (\mathcal{L}, \mid\!\sim_{\mathcal{G}3})$ (for the definition of n-dimensional Gentzen systems, see the abstract *Algebraizable Gentzen systems* in this volume) such that:

(1) This axiom system has the cut-elimination property.

(2) $\mathcal{G}_3$ is algebraizable in the sense of J. Rebagliato and V. Verdú ([3] and [4]), and the variety generated by A_3 is the equivalent variety semantics for $\mathcal{G}_3$.
(3) $\mathcal{G}_3$ has the deduction Theorem (in the sense of [3], and [4]).
(4) $\mathcal{G}_3$ allows to obtain $\mathcal{S}_3$ in the following way:
for all $\{\varphi_1, \dots, \varphi_n, \varphi\} \subset \mathcal{F}m_{\mathcal{L}}\varphi_1, \dots, \varphi_n \vdash_3 \varphi$ iff $\emptyset \mid\!\sim_3 \varphi_1, \dots, \varphi_n \mid \varphi_1, \dots, \varphi_n \mid \varphi$.

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We introduce and investigate extensions of the propositional modal and temporal languages with simple syntactic gadgets called *local names* and *reference pointers*. These provide an efficient mechanism for implicit designation of a semantic entity (possible world, time instant etc.) within a formula by inserting a "local name", and then making references to that entity from elsewhere in that expression by using "reference pointers".

A simple scheme within a formula φ looks like this : $\varphi = \dots \downarrow (\dots \uparrow \dots \uparrow \dots) \dots$, possibly iterated. Semantically (in Kripke semantics) it means the following when the evaluation process of the formula φ reaches the subformula $\downarrow (\dots \uparrow \dots \uparrow \dots)$ the actual point (at which the evaluation is being done at that moment) of the model receives the local name $\uparrow$ and thus becomes the denotation for every "free" (not in the scope of another $\downarrow$) occurrence of the reference pointers $\uparrow$ within the subformula.

The formal languages thus extended come considerably closer to natural languages since the reference pointers play the role of adverbs ("there", "then"), so much missed in the modal and temporal languages.

Adding several reference sets "local name – pointers" we obtain hierarchies of increasingly expressive, very powerful modal and temporal languages. Eventually they converge to a common limit which covers the Π^1_1-fragment of the monadic second-order language for relational structures.

We discuss the expressive power of the languages in these hierarchies, axiomatize their logics and prove undecidability of the satisfiability problem in them.

Finally we consider extensions of these languages by means of *locally definable operators* and show a general completeness result for these extensions.

► L. GORDEEV, *Cut free formalization of predicate logic with finitely many variables*.
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Let $\mathcal{L}_n$ be $\{\top, \bot, \vee, \wedge, \neg, \forall, \exists\}$-language of the 1-order predicate logic without function symbols and equality ($\neg$ acting on atomic level), whose variables are v_i for $0 \leq i \leq n \leq \omega$. Let $\mathbf{RPC}_n$ be the term rewriting system in $\mathcal{L}_n$ with the following rules (*elementary reductions*).

$$
\begin{array}{llll}
(\tau) & \Delta \vee L \vee \Pi \vee \neg L & \longrightarrow & \top \\
(\iota) & A \vee \top & \longrightarrow & \top \quad \text{and} \quad \top \vee A \longrightarrow \top \\
(\hat{\iota}) & A \wedge \top & \longrightarrow & A \quad \text{and} \quad \top \wedge A \longrightarrow A \\
(\alpha) & A \vee (B \vee C) & \longrightarrow & A \vee B \vee C \\
(\delta) & (A \wedge B) \vee C & \longrightarrow & (A \vee C) \wedge (B \vee C) \\
 & & & \quad \text{and} \quad C \vee (A \wedge B) \longrightarrow (C \vee A) \wedge (C \vee B) \\
(\pi) & \Delta \vee \forall x A \vee \Pi & \longrightarrow & \Delta \vee \forall x A \vee A[\div x] \vee \forall y(\Delta[\div y] \\
 & & & \quad \vee A[x \div y][x/y] \vee \Pi[\div y]) \vee \Pi \\
(\sigma) & \Delta \vee \exists x A \vee \Pi & \longrightarrow & \Delta \vee \exists x A \vee A[x/t] \vee \exists y(\Delta[\div y] \\
 & & & \quad \vee A[x \div y][x/y] \vee \Pi[\div y]) \vee \Pi
\end{array}
$$

(L := literals; Δ, Π := "long" disjunctions or $\emptyset$; A, B, C := formulas; x, y := variables; t := terms; $[\div x], [x \div y], [x/t]$:= "deletion", "subtraction", "substitution" which do not increase the length.) A is called *derivable* in $\mathbf{RPC}_n$ ($\mathbf{RPC}_n \vdash A$) iff there is a reduction chain starting with A and ending by $\top$. (*Reductions* arise by applying elementary reductions to arbitrary subformulas.)

THEOREM 1. $\mathbf{RPC}_n \vdash A$ *iff A is derivable in the standard modus ponens predicate calculus in the presence of at most n distinct variables.*

THEOREM 2. *There is a polynomial embedding* $\mu^* : \mathcal{L}_\omega \longrightarrow \mathcal{L}_3$ *such that for any formula A of* $\mathcal{L}_\omega$, $\mu^*(A)$ *is a formula of* $\mathcal{L}_3$*, and for every* $n > 3$*:*

$$\mathbf{RPC}_n \vdash \mu^*(A) \ \textit{ iff } \ \mathbf{RPC}_\omega \vdash A \ \textit{ iff } \ \models A$$

In particular, A is valid iff $\mu^*(A)$ *is derivable in* $\mathbf{RPC}_4$.

Theorem 2 is also true of the particular "minimal" language $\mathcal{L}_3$ containing only one binary predicate and no individual constant.

THEOREM 3. *For each* $n > 2$ *there is a sentence of the minimal language* $\mathcal{L}_3$ *(as above) which is derivable in* $\mathbf{RPC}_{n+1}$*, but not in* $\mathbf{RPC}_n$.

If $\mathcal{L}_n$ also includes the equality "$\equiv$", the correlated calculus with equality, $\mathbf{RPCE}_n$, extends $\mathbf{RPC}_n$ (conservatively) by the following elementary reductions.

$$
\begin{array}{llll}
(\varepsilon) & x \equiv x & \longrightarrow & \top \\
(\gamma) & x \not\equiv y & \longrightarrow & x \not\equiv y \vee y \not\equiv x, \quad if \quad x \neq y \\
(\lambda) & \Delta \vee x \not\equiv y \vee \Pi & \longrightarrow & \Delta \vee \Delta[x/y] \vee x \not\equiv y \vee \Pi[x/y] \vee \Pi, \quad if \quad x \neq y
\end{array}
$$

THEOREM 4. $\mathbf{RPCE}_n \vdash A$ *iff A is derivable in the standard modus ponens predicate calculus with equality in the presence of at most n distinct variables.*

COROLLARY 5. *The theorems of Tarski's Relation Algebra are, modulo the canonical interpretation, the three-variable sentences derivable in* $\mathbf{RPCE}_4$.

► WILLIAM G. HANDLEY, *Nondeterministic summation mod* $\mathcal{T}_3$*, the semigroup of functions on* $\{0, 1, 2\}$.

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Deterministic summation *modulo* an arbitrary semigroup is an extension of the schemes for bounded sum and bounded product used to define the class of Csillag-Kalmar elementary functions. For an arbitrary class $\mathcal{R}$ of relations on natural numbers, one has the closure $\overline{\mathcal{T}_n} - \Delta_0(\mathcal{R})$ under deterministic summation *mod* $\mathcal{T}_n$, the semigroup of functions with domain and codomain $\{0, 1, \ldots, n-1\}$.

Using P. Clote's definition of nondeterministic summation, which extends this even further, we consider the closure $\mathrm{N}\overline{\mathcal{T}_n} - \Delta_0(\mathcal{R})$ and show that

$$\mathrm{N}\overline{\mathcal{T}_3} - \Delta_0(\mathcal{R}) = \overline{\mathcal{T}_3} - \Delta_0(\mathcal{R})[= \overline{\mathcal{S}_3} - \Delta_0(\mathcal{R})].$$

This resolves a question in [1]. The case $n = 4$ remains open.

We also show that if $\mathcal{G}$ is any finite group then

$$\mathrm{N}\overline{\mathcal{G}} - \Delta_0(\mathcal{R}) = \overline{\mathcal{G}} - \Delta_0(\mathcal{R}).$$

[1] P. Clote, *Stack register machines: non-determinism, the linear time hierarchy, and uniform boolean circuit classes*, (Extended Abstract), 1993.

► KOSTAS HATZIKIRIAKOU, *WKL_0 and Stone's separation theorem for convex sets*.

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This paper is a contribution to the program of Reverse Mathematics; see [2]. We prove that Stone's Separation Theorem for convex sets (in countable vector spaces over a countable ordered field) is provable in WKL_0. Moreover, we prove that a version of it is equivalent to Weak König's Lemma. As a corollary, we obtain a result due (independently) to I. Kalantari and R. Downey, i.e., there exist two r.e. convex subset of V_∞ which are disjoint but they cannot be separated by a recursive convex subset of V_∞ whose complement is also convex; see [1]. The main theorems of the paper are:

Theorem 1 (WKL_0). *Let V be a countable vector space over a countable ordered field C. Let C be a Σ^0_1 convex subset of V such that $O \notin C$. Then, there exists a hypercone $K(o)$ of V such that $C \subseteq K(o)$.*

Theorem 2 (WKL_0). *Let V be a countable vector space over a countable ordered field F. Let C, D be disjoint convex sets. Then, there exists a convex set H such that V/H is convex, $C \subseteq H$ and $D \subseteq V/H$.*

Theorem 3 (RCA_0). *The following are equivalent.*

(1) *Weak König's Lemma.*

(2) *Let V be a countable vector space over a countable ordered field F. Let C, D be disjoint Σ^0_1 convex sets. Then, there exists a convex set H such that V/H is convex, $C \subseteq H$ and $D \subseteq V/H$.*

[1] R. Downey, *Some remarks one a theorem of Iraj Kalantari concerning convexity and recursion theory*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 30 (1984), pp. 303–312.

[2] H. Friedman, S. G. Simpson, and R. Smith, *Countable algebra and set existence axioms*, ***Annals of Pure and Applied Logic***, vol. 25 (1983), pp. 141–181, Addendum in vol. 27 (1985) pp. 318–320.

► LAURI HELLA AND JOUKO VÄÄNÄNEN AND DAG WESTERSTÅHL, *Definability of polyadic lifts of generalized quantifiers*.

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We study generalized quantifiers on finite structures. The best understood among such quantifiers are the monotone simple monadic ones. With every function $f : \omega \to \omega$ we can associate a monotone simple monadic quantifier Q_f by letting $Q_f x\phi$ say "there are at least $f(n)$ elements x satisfying ϕ, where n is the size of the universe". Our results hold also for the relativized version $Q'_f xy\phi\psi$ which says "of the n elements that satisfy ψ at least $f(n)$ satisfy ϕ". We say that f is *unbounded*, if $\forall m \exists n(m \leq f(n) \leq n - m)$, otherwise *bounded*. Canonical examples of unbounded functions are $\lfloor \frac{n}{2} \rfloor$, $\lfloor \sqrt{n} \rfloor$ and $\lfloor \log(n) \rfloor$, and of bounded ones $f(n) = 5$ for even n and $f(n) = n - 6$ otherwise.

In this paper we study *polyadic lifts* of such quantifiers Q_f. Unlike monadic quantifiers, that only talk about relative numbers of elements, polyadic quantifiers talk about properties of tuples. To say that a graph contains so and so many vertices of degree 3 one uses a monadic quantifier of the form Q_f. But to say that a graph contains a clique of such and such size, one needs a polyadic quantifier. *Lifting* is a way of transforming monadic quantifiers Q into polyadic ones $Lift(Q)$. We study the following two particular lifts:

The *k-ary Ramsey* quantifier defined by Q_f is the quantifier $Ram^k(Q_f)x_1 \ldots x_k\phi$ which holds in a structure of size n if and only if there is a subset X of the universe of the model such that $|X| \geq f(n)$ and every k-tuple of different elements from X satisfies ϕ. So with $Ram^2(Q_f)$ we can say a graph has a clique of size $\geq f(n)$.

The *k-ary Branching* of Q_f is the quantifier $Br^k(Q_f)x_1 \ldots x_k\phi$ which holds in a structure of size n if and only if there are sets $X_1, \ldots, X_k$ of size $\geq f(n)$ so that every k-tuple in $X_1 \times \ldots \times X_k$ satisfies ϕ. With $Br^2(Q_f)$ we can say "at least $f(n)$ boys in my class and $f(n)$ girls in your class have dated each other this year".

A natural question to ask about a lift is, whether $Lift(Q)$ is definable in $\mathcal{L}_{\omega\omega}(Q)$ or not, that is, whether the lift brings anything new at all. We have the following general answer to this question:

(1) $Ram^k(Q_f)$ is definable in $\mathcal{L}_{\omega\omega}(Q_f)$ if and only if f is bounded.
(2) $Br^k(Q_f)$ is definable in $\mathcal{L}_{\omega\omega}(Q_f)$ if and only if f is bounded.

Actually we prove more: if f is unbounded, then $Br^k(Q_f)$ is not definable in the extension of first order logic by any monadic quantifiers, and $Ram^{k+1}(Q_f)$ is not definable in the extension of $\mathcal{L}^{\omega}_{\infty\omega}$ by any finite number of k-ary quantifiers. We discuss also other lifts.

► SACHIO HIROKAWA, YUICHI KOMORI AND IZUMI TAKEUTI, *A reduction rule for Peirce's formula*.

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Implicational theorems in classical logic are characterized by

$$S : (\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma, K : \alpha \to \beta \to \alpha$$

and *Pierce's formula* $P : ((\alpha \to \beta) \to \alpha) \to \alpha$. The first two determine the intuitionistic logic and have the reduction rules $Sxyz \Rightarrow xz(yz)$ and $Kxy \Rightarrow x$. This two reduction rules correspond to the normalization of proofs in Natural Deduction System, hence to the $\beta-$reduction of typed $\lambda-$terms [1, 3]. What is a reduction rule for Pierce's formula? What kind of equivalence of proofs is derived from such a reduction rule? In other words, how do we extend the Curry-Howard isomorphism [2] to the classical logic ?

We introduce a reduction rule ($P-$reduction).

$$x^{\alpha \to \beta}(P^{((\alpha \to \beta) \to \alpha) \to \alpha} y^{(\alpha \to \beta) \to \alpha}) \stackrel{P}{\Longrightarrow} x(yx).$$

It represents the following transformation of proof figures in classical logic which eliminates superfluous occurrence of Pierce's axiom.

$$\dfrac{\begin{matrix}\vdots D_1 \\ \alpha \to \beta\end{matrix} \qquad \dfrac{((\alpha\to\beta)\to\alpha)\to\alpha \quad \begin{matrix}\vdots D_2 \\ (\alpha\to\beta)\to\alpha\end{matrix}}{\alpha}}{\beta} \quad \overset{P}{\Longrightarrow} \quad \dfrac{\begin{matrix}\vdots D_1 \\ \alpha \to \beta\end{matrix} \qquad \dfrac{\begin{matrix}\vdots D_2 \\ (\alpha\to\beta)\to\alpha\end{matrix} \quad \begin{matrix}\vdots D_1 \\ \alpha\to\beta\end{matrix}}{\alpha}}{\beta}$$

We prove the following theorem which says that all the proofs of the same formula are equivalent with respect to the $\beta P-$convertibility.

LEMMA. $x^{\alpha\to\alpha}y^{\alpha} =_{\beta P} x(xy)$.

THEOREM. $A^{\alpha} =_{\beta P} B^{\alpha}$.

[1] J. R. HINDLEY and J. P. SELDIN, ***Introduction to combinators and lambda-calculus***, Cambridge University Press, London, 1986.

[2] W. A. HOWARD, *The formulae-as-types notion of construction*, ***To H. B. Curry, essays on combinatory logic, lambda calculus and formalism*** (Hindley and Seldin, editors), Academic Press, 1980, pp. 479–490.

[3] D. PRAWITZ, ***Natural deduction***, Almqvist and Wiksell, Stockholm, 1965.

► TAKAO INOUÉ, *A generalized Gödel's Theorem*.

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Let X be classical predicate (propositional) logic without equality **CQC (CPC)**.

Let Y be intuitionistic predicate (propositional) logic without equality **IQC (IPC)**.

For any atomic formula p of Y, we define two translations $id_{\delta\{p\}}$ and $id_{\iota\{p\}}$ from X to Y as follows: *for any formula A of X, $id_{\delta\{p\}}(A) = id_{\iota}(A) \vee p$ and*

$$id_{\iota\{p\}}(A) = \begin{cases} id(A) \supset p. \supset p & \text{if } p \neq \perp \text{ and } p \text{ does not occur in } id(A); \\ \neg\neg id(A) & \text{if } p = \perp; \\ id(A) & \text{otherwise} \end{cases}$$

where id is identity translation.

A pair $(\tilde{p}, \sigma)$ is said to be a *pre-G-sequence of Y with length n* if it satisfies the following conditions (1) and (2):

(1) $\tilde{p}$ is an ordered finite sequence $p_1, p_2, \ldots, p_n (n \geq 1)$ of atomic formulas of Y;

(2) σ is a mapping from $\{1, 2, \ldots, n\}$ to $\{\delta, \iota\}$.

A pre-G-sequence $(\tilde{p}, \sigma)$ of Y with length n, say $\tilde{p} = (p_1, p_2, \ldots, p_n)$ is said to be a *non-$\perp$-sequence of Y with non-$\perp$-index i*, if $\exists 1 \leq j \leq n(p_j \neq \perp)$ holds with $i = max\{j : p_j \neq \perp, 1 \leq j \leq n\}$, otherwise it is called a *$\perp$-sequence of Y*.

A pre-G-sequence $(\tilde{p}, \sigma)$ of Y with length n, say $\tilde{p} = (p_1, p_2, \ldots, p_n)$ is said to be a *G-sequence of Y*, if it satisfies one of the following conditions (a) and (b):

(a) it is a $\perp-$ sequence of Y ;

(b) it is a non-$\perp$-sequence of Y with non-$\perp$-index i such that if $i \geq 2$, then $\forall 1 \leq j \leq i - 1[p_j = \perp \bigwedge \sigma(j) = \iota. \supset \neg\exists j + 1 \leq k \leq i(p_k \neq \perp \bigwedge \sigma(k) = \delta)]$.

Let $(\cdot)^{\circ}$ be Gödel translation from X to Y. For any pre-G-sequence $(\tilde{p}, \sigma)$ of Y with length n, say $\tilde{p} = (p_1, \ldots, p_n)$, we define a translation $\zeta_{\sigma}(\tilde{p})$ from X to Y as follows :

for any formula A of X,

$$\zeta_{\sigma}(\tilde{p})(A) = id_{\sigma(1)\{p_1\}}(id_{\sigma(2)\{p_2\}}(\cdots(id_{\sigma(n)\{p_n\}}(A^{\circ}))\cdots)).$$

Let $\mathcal{G}$ be an equivalence class $\{\zeta_{\sigma}(\tilde{p}) : (\tilde{p}, \sigma) \text{ is a G-sequence of} Y\}/_{\equiv_c}$, which is called a *generalized Gödel translation fromX to Y,* where $\equiv_c$ is C-equivalence (for (non-)C-equivalence, see [2]).

MAIN RESULTS: For any pre-G-sequence $(\tilde{p}, \sigma)$ of Y, $\zeta_\sigma(\tilde{p})$ is an embedding of X in Y iff $(\tilde{p}, \sigma)$ is a G-sequence of Y. (In particular, for any $\perp$-sequence $(\tilde{p}, \sigma)$, $(\cdot)^\circ$ and $\zeta_\sigma(\tilde{p})$ are C-equivalent.) The cardinality of $\mathcal{G}$ is $\aleph_0$. We give a necessary and sufficient condition of the non-C-equivalence of elements of $\mathcal{G}$.

The full description of the results and their full proofs are given in [1].

[1] T. INOUÉ, *Corrections and additions to my paper "A note on unprovability-preserving sound translations", part I; part II; part III*, Forthcoming. It is possible that the titles of these papers will be changed.

[2] ———, *A note on unprovability-preserving sound translations*, ***Logique et Analyse***, vol. 33 (1990), pp. 243–257, (This paper was published in December, 1993).

► ALEXANDRE IVANOV, *Finite covers of ω-categorical structures and cohomology*.
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Let N be a countably categorical transitive structure and F be a finite set. A structure $M = N \cup (N \times F)$ with the projection $N \times F \rightarrow N$ and the N- relations is a *cover* of N if the natural homomorphism from $Aut(M)$ into $Aut(N)$ is surjective. A cover M is split if it admits a covering expansion M' with trivial *kernel* (the kernel of $Aut(M') \rightarrow Aut(N)$). M is strongly split if there is a partition of $N \times F$ by some its transversals $P_1, \dots, P_k (\forall x \in N \exists! f \in F : (x, f) \in P_i)$ such that the expanded structure $(M, P_1, \dots, P_k)$ is a cover of N too.

We classify all finite covers of highly homogeneous structures. In particular, we prove that they can be non-split and can be split but not strongly split. We show that the Cameron's cohomology approach to two-graphs can be applied to construction of non-split covers of some wide class of structures including some highly homogeneous ones. We also apply these ideas to construction of *universal* covers. It is shown that some natural assumptions on the group of strong automorphism imply a complete characterization of finite covers.

► JIHAD JAAM, *Coloriages cycliques pour les hypergraphes complets associés aux nombres de Ramsey classiques ternaires*.
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Soit X un ensemble de n éléments, et soit $\mathcal{P}_r(X)$ l'ensemble des parties (une famille) à r éléments de X $\left(|\mathcal{P}_r(X)| = \binom{n}{r} = \frac{\mathrm{n}!}{\mathrm{r}!\,(n-r)!}\right)$. Le couple $H = \left(X, \mathcal{P}_r(X)\right)$ constitue un*hypergraphe complet de rang r et d'ordre n*. Les éléments de X et de $\mathcal{P}_r(X)$ désignent respectivement les sommets et les arêtes de H. Chaque arête de H est donc un ensemble de cardinal r. Un tel hypergraphe est appelé une*n-clique de rang r* et se note K_n^r. Nous allons introduire le théorème général de Ramsey sous forme de coloration des arêtes de H. Considérons m nombres entiers $k_1, k_2, \dots, k_m > r$ et m couleurs différentes $\{1, 2, \dots, m\}$. On définit une $(k_1, k_2, \dots, k_m; r)$*-coloration* comme étant la fonction qui affecte à chaque arête de K_n^r une seule couleur parmi m couleurs possibles, de sorte qu'aucune clique de k_i sommets et de rang r n'ait toutes ses arêtes de couleur i. Cette fonction est appelée*une bonne coloration des arêtes de* K_n^r. Un tel hypergraphe est dit $(k_1, k_2, \dots, k_m; r)$*-colorable.*

Le théorème de Ramsey (1930) affirme l'existence d'un plus petit entier fini $R(k_1, k_2, \dots, k_m; r)$ tel que tout hypergraphe complet de rang r avec un nombre de sommets $n \geq R(k_1, k_2, \dots, k_m; r)$ n'admette aucune bonne $(k_1, k_2, \dots, k_m; r)$-coloration pour ses arêtes. Ce nombre s'appelle*le nombre de Ramsey classique de rang r en* $k_1, k_2, \dots, k_m$. En d'autres termes, tout coloriage des arêtes de K_n^r avec $n = R(k_1, k_2, \dots, k_m; r)$, contient au moins une k_i-clique monochrome de rang r et de couleur i.

L'évaluation de chaque nombre de Ramsey est un problème NP-difficile. On se contente très souvent d'améliorer les bornes inférieures et supérieures de ces nombres. Ainsi, pour améliorer leur minorant, on cherche un hypergraphe K_n^r avec n le plus grand entier possible, pour lequel il existe une bonne $(k_1, k_2, \dots, k_m; r)$-coloration. De même, pour améliorer leur majorant, on étudie le plus petit entier n $\Big(n \geq R(k_1, k_2, \dots, k_m; r)\Big)$, pour lequel il n'existe aucune bonne $(k_1, k_2, \dots, k_m; r)$-coloration. On construit très souvent les bonnes colorations on se servant principalement des propriétés des coloriages cycliques pour les graphes, introduites par J.G. Kalbfleisch en 1966. Nous introduisons dans cet article, les coloriages cycliques pour les hypergraphes.

Nous proposons quelques algorithmes d'optimisation stochastique pour évaluer (ou approximer) certains nombres de Ramsey classiques binaires et ternaires. Nous montrons que les coloriages cycliques existent non seulement pour les graphes complets associés aux nombres de Ramsey classiques binaires mais également pour les hypergraphes complets associés aux nombres de Ramsey classiques ternaires. Nous utilisons pour cela la notion de longueur des arêtes des hypergraphes. Nous démontrons quelques propriétés d'inexistence de certains coloriages cycliques pour ces graphes et hypergraphes.

Mots clés : coloriage cyclique, nombre de Ramsey, graphe, hypergraphe.

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Storrs McCall (***Aristotle's Modal Syllogisms***, North-Holland, Amsterdam, 1963) gives a deductive system, Q-L-X-M, that is intended to organize Aristotle's judgments about the validity of assertoric, apodeictic, and contingent syllogisms. Q-L-X-M is shown to generate theses that McCall overlooked. All syllogisms of the form $(QAbc \rightarrow (LAab \rightarrow x))$ and $(LAbc \rightarrow (QAab \rightarrow x))$ are theses, though Aristotle claims that Barbara-QLX and Barbara-LQX are invalid. That McCall overlooked no other theses is shown by means of a formal semantics for the system. Since there are 29 discrepancies with Aristotle's judgments about the validity of "basic syllogisms." An alternative to Q-L-X-M is proposed that generates fewer discrepancies. A natural and Aristotelian semantics is given for the latter system, using some of the semantic conditions given by Fred Johnson (*Models for Modal Syllogisms*, ***Notre Dame Journal of Formal Logic***, vol. 30 (1989) pp. 271–284 and *Modal Ecthesis*, ***History and Philosophy of Logic***, vol. 14 (1993), pp. 171–182) and S. K. Thomason (*Semantic Analysis of the Modal Syllogistic*, ***Journal of Philosophical Logic***, vol. 22 (1993), pp. 111–128).

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Bien qu'elles se soient appuyées sur des techniques inouïes de l'intelligence artificielle, les mathématiques, en offrant une interprétation empirio-éclectique aux phénomènes du chaos et du désordre, restent tributaires de façon inductive, de la logique classique. Dès que l'on étudie de tels phénomènes, les principes aristotéliciens de l'identité, du tiers exclu générateurs des structures d'ordre, deviennent caducs. Cela nécessite, par conséquent de nouvelles méthodes, un nouveau langage et surtout une nouvelle logique. La pensée du chaos soulève une question plus radicale liée au renouvellement du paradigme de la rationalité mathématique dont les injonctions sont graves. L'adoption de nouveaux principes on ne peut plus stoïciens que voici:

le principe d'altérité: A une proposition $e(c)$ (lire, "e contre c") s'urgit la proposition *alterne* $c(e)$; on écrit $e(c)$ P $c(e)$ (lire, $\ll e(c)$ *puis* $c(e) \gg$) ;

le principe de densité: A deux propositions *alternes* $e(c)$ et $c(e)$, on peut trouver une tierce proposition h qui est l'*objet* de l' *altérité* telle que [*e,c*] (h) [*c,e*] (lire, "*e*, *c* contre *h* et *c*, *e* pour *h* ") ;

le principe de mobilité: Deux propositions sont *en mouvement* $e(c)$ et $(e)c$ (lire"*c* pour *e*") ou forment *une controverse* quand elles s'opposent de telle manière que la proposition alterne de $e(c)$ dénie la propriété exprimée par $(e)c$; on écrit $e(c) \parallel (e)c$ (lire $\ll e(c)$ *pendant que* $(e)c \gg$); grâce à ces principes, nous pouvons comprendre le déroulement des *situations chaotiques* au niveau logique.

Nous avons élaboré une fondation des *mathématiques chaotiques*. La *logique chaotique avec altérité* est la logique interne aux *mathématiques chaotiques*. La *théorie des modes* est la sémantique de la logique chaotique. *La théorie axiomatique des espèces*, avec ses axiomes propres est un modèle *concurrent* à théorie des ensembles selon Zermelo-Frankael. Les *Greegébres* sont une extension de la *théorie des espèces*. Pour ce qui concerne des *Espaces mbuuka*, ce sont des espaces non topologiques oû l'idée de la forme ne se confond pas avec l'idée de l'ordre. Chaque théorie fondationnelle est une méthode conduisant vers la compréhension du chaos. Ces cinq théories s'interpénètrent comme les cinq doigts de la main, au point de former une quintessence pour les "mathématiques à venir."

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Introducing double meaning for the membership relation makes possible to return to ideas of the original Frege proposal and base set theory on a single axiom-schema of comprehension.

The language of the presented system, denoted by IC2, is that of the first order predicate calculus with equality, supplemented by two binary predicate symbols $\in$ and $\mathcal{E}$. Any formula $\varphi(x)$ making no use of the symbol $\mathcal{E}$ is called *normal*. The *dual* of this is the formula $\overline{\varphi}(x)$ obtained from $\varphi(x)$ just by replacing each occurrence of $\in$ by that of $\mathcal{E}$. Let $R(y)$ abbreviate $\forall x(x \in y \leftrightarrow x \mathcal{E} y)$ (which means that for the set y both the membership relations coincide). A set y having this property or satisfying either $\forall x(x \in y \rightarrow R(x))$ or $\forall x(x \mathcal{E} y \rightarrow R(x))$ will be called *regular*. The nonlogical axioms of IC2 are the following.

I. AXIOM OF EXTENSIONALITY. *For all sets x,y,* $\forall z(z \in x \leftrightarrow z \mathcal{E} y) \rightarrow x = y$.

II. AXIOM OF COMPREHENSION. *For all regular sets* $x_1, \ldots, x_n$, $\exists y \forall x(x \mathcal{E} y \leftrightarrow \varphi(x) \wedge x \in y \leftrightarrow \overline{\varphi}(x))$, *where* $\varphi(x)$ *is any normal formula in which no other variable than* $x, x_1, \ldots, x_n$ *occurs free, and* $\overline{\varphi}(x)$ *is its dual.*

We show that the universe IC2 contains all (regular) sets whose existence is provided by the axioms of ZF, and therefore, IC2 allows as trouble-free development of mathematics as ZF does. No familiar paradoxes can be reproduced in IC2. The system contains a mechanism preventing reproducing antinomies. However, the following is still open:

Problem. *Is there any consistency proof of IC2 relative to a presently accepted system of set theory?*

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We show how a new monotone version of modified realizability can be used to extract bounds on $\bigvee w^2$ in sentences $G :\equiv \bigwedge u^1 \bigwedge i \leq_\gamma tu(\neg C \rightarrow \bigvee w^2 D)$ which are proved in (sub-)systems $\mathcal{T}_i^\omega$ of intuitionistic arithmetic E-HA$^\omega$ (in the language of all finite types) **plus arbitrary non-constructive analytical sentences** having the form $F :\equiv \bigwedge x^\delta(A \to \bigvee y \leq_\rho sx\neg B)$, where A, B, C, D are **arbitrary** formulas $\in \mathcal{L}(\mathcal{T}_i^\omega)$ (such that G, F are closed), γ, δ, ρ are arbitrary finite types, s, t are closed terms and $x_1 \leq_{\rho(\tau)} x_2 :\equiv \bigwedge y^\tau(x_1 y \leq_\rho x_2 y)$.

More precisely we prove results of the following type:

THEOREM. *If $\mathcal{T}_i^\omega + F + IP_\neg(+AC) \vdash \bigwedge u^1 \bigwedge v \leq_\gamma tu(\neg C \to \bigvee w^2 Dw)$, then there exists an effectively a closed term Φ of $\mathcal{T}_i^\omega$, such that*

$$\mathcal{T}^\omega + (b-AC) + F(+AC) \vdash \bigwedge u \bigwedge v \leq tu(\neg C \to \bigvee w \leq_2 \Phi u Dw),$$

where AC denotes the schema of choice in the language of functionals of finite types, and

$(b-AC)$	:	$\bigwedge Z^{\tau\rho}(\bigwedge x^\rho \bigvee y \leq_\tau Zx\, A(x,y,Z) \to \bigvee Y \leq_{\tau\rho} Z \bigwedge xA(x, Yx, Z))$,
$IP_\neg$	:	$(\neg A \to \bigvee y^\sigma B) \to \bigvee y^\sigma(\neg A \to B)$ (y not free in A),
$\mathcal{T}^\omega$	:	$\mathcal{T}_i^\omega +$ classical logic.

EXAMPLES FOR LEMMAS F are:

(1) attainment of the maximum of $f \in \mathcal{C}([0,1]^n, \mathbb{R})$ on $[0,1]^n$,
(2) mean value theorems,
(3) Cauchy-Peano existence theorem,
(4) Brouwer's fixed point theorem for $\mathbb{R}^n$,
(5) existence of best approximations,
(6) "lesser limited principle of omniscience",
(7) weak König's lemma WKL,
(8) comprehension for negated formulas : $\bigvee \Phi \leq_{0\rho} \lambda x^\rho.1 \bigvee y^\rho(\Phi y =_0 0 \leftrightarrow \neg A(y))$.

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Generalized Pascal triangles /GPT/ are mappings of some cofinite subsets of $\mathbb{N} \times \mathbb{N}$ into finite sets associated to some finite algebras analogously as the classical Pascal triangle can be associated to the (infinite) algebra $\langle \mathbb{N}; +, 0\rangle$. More formally, for every finite algebra $\mathcal{A} = \langle \mathbf{A}; *, o\rangle$ (with $\mathbf{A} \subseteq \mathbb{N}$ to avoid some technical problems) and every word $w = w_0 \dots w_{|w|-1} \in \mathbf{A}^+$ the function $G = \text{GPT}(\mathbf{A}, w)$ will be defined by

$$G(x,y) = \begin{cases} \text{undefined} & \text{if } x+y <\mid w \mid -1, \\ w_x & \text{if } x+y =\mid w \mid -1, \\ \text{o} * G(0, y-1) & \text{if } x = 0,\ y \geq\mid w \mid, \\ G(x-1, 0) * \text{o} & \text{if } y = 0,\ y \geq\mid w \mid, \\ G(x-1, y) * G(x, y-1) & \text{if } x+y \geq\mid w \mid,\ x > 0,\ y > 0. \end{cases}$$

For example, if $\mathbb{Z}_n$ is the additive group modulo n then $B_n = \text{GPT}(\mathbf{Z}_n, 1)$ is the Pascal triangle modulo n; we have $B_n(x,y) = \begin{pmatrix} x+y \\ x \end{pmatrix}$ MOD n.

Theories of various structures related to GPT and classes of such structures will be investigated. Every considered structure will have the base set $\mathbb{N}$, it will contain a GPT and possibly some additional operations or relations, e.g. the successor function $\mathbf{s}$. The elementary theories of such structures can be undecidable already for $\mid \mathbf{A} \mid= 3$; this bound is exact, and does not depend on $\mid w \mid$ and the presence of the additional operation $\mathbf{s}$ or $+$. It is diminished to 2 if the additional operation is $\times$ or the additional relation is the divisibility relation or the set of squares as a unary relation; the necessary GPT for that is B_2. Analogously it is diminished to 2 if theories of classes of structures are considered.

The addition $+$ is not definable in any structure $\langle \mathbb{N}; B_n \rangle$, n prime. However, it is definable in every such structure if $n > 0$ is composite. Moreover, if $n > 0$ has two distinct prime divisors, then $\times$ is definable, too, and hence the elementary theory of $\langle \mathbb{N}; B_n \rangle$ is undecidable.

The addition is definable in the structures $\langle \mathbb{N}; B_n, \mathbf{s} \rangle$ n prime, and the defining formula can be chosen independently on n. The elementary theory of every structure $\langle \mathbb{N}; B_n, + \rangle$ is decidable for every prime n. In the proof a reduction to weak monadic second order (WMSO) theory of successor is used.

The operations $+, \times$ are not WMSO definable in $\langle \mathbb{N}; B_2 \rangle$. Nevertheless, the WMSO theory of the structure $\langle \mathbb{N}; B_2 \rangle$ is undecidable; the elementary theory of $\langle \mathbb{N}; +, \times \rangle$ can be interpreted in it. The same hold for every prime instead of 2.

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Getmanova's logic uses some rational numbers as logical values [1]. But it is convenient to use much more rational numbers for this aim. More exactly we can take any complete closed set of rational numbers which is symmetrical about zero. For such set we can extend Gotmanova's logic to the predicates. Instead of "The exist X such that A" we can use "The supremum A under all X". Instead of "For all X we have A" we can use "Infimum A under all X". We will be interested only in such formulas which logical values are not less than zero (not less than $1/2$ in Getmanova's logic). The true values are greater than zero. The false values are less than zero. And zero is a paradox as logical value. The operation of negation is the minus operation. The conjunction is the minimum of all arguments. Disjunction is maximum of all arguments. Sequent axioms have the form $XyZ \to VyW$ or $W \to XeY$, where $e \geq 0$, or $XdY \to W$, where $d \leq 0$. The rules are defined as the usually in sequent calculus.

THEOREM 1. *In the proposed logical system the axioms are not false and the rules preserve not falsity.*

THEOREM 2. *Every formula without quantifier which value is not less than zero can be proved in the proposed calculus.*

THEOREM 3. *Any propositional formula without logical constants is not false iff it is a tautology of classic propositional logic.*

[1] A. GETMANOVA, ***Logic***, Progress Publishers, Moscow, 1989.

► T. G. KUCERA, *On the structure of indecomposable totally transcendental modules*.

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There are many interesting problems here, as a significant special case is that of indecomposable injective right modules over a (right) Noetherian ring, where we find some important longstanding problems of purely algebraic interest. In the last few years ideas originating in model theory have begun to provide some new insight into what were apparently strictly algebraic questions. In particular, both the elementary cycle series introduced by I. Herzog [1] and my regular hierarchy [2] are well defined and exhaustive in any indecomposable totally transcendental module, whereas the usual algebraic analysis of indecomposable injectives ("fundamental series") requires additional assumptions.

I will report on some recent work on the algebraic and model-theoretic properties of these two series. In particular, I will discuss possible applications to such algebraic problems as non-commutative localization theory and Jacobson's conjecture.

[1] I. HERZOG, *Elementary duality of modules*, ***Transactions of the American Mathematical Society*** (1994).

[2] T. G. KUCERA, *The regular hierarchy in modules*, ***Communications in Algebra***, vol. 12 (1993), pp. 4615–4628.

► KANAT KUDAIBERGENOV, *Atomic models of locally modular theories of finite rank*.
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Some conditions for LMFR (i.e., locally modular of finite rank) extra stable theories to have an atomic model over a set are indicated.

Also, some results on the existence of atomic models of different powers for such theories are obtained.

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In [5], L. van den Dries, A. Macintyre and D. Marker describe a method for the construction of nonarchimedean exponential fields which are elementarily equivalent to $(\mathbb{R}, \exp)$. In this note, we will give an alternative method which demands less technical effort. We eliminate one of the two limit processes used in [5], by a direct construction of fields with (non-surjective) logarithms. This is based on the point of view as developed in [2] and [1].

Indeed, we shall construct a field K_ω as the union of a chain $K_\omega = \bigcup_{n\in\mathbb{N}} K_n$, where every K_n is a power series field with real coefficients and exponents in a divisible ordered abelian group G_n (which is the value group of the natural valuation v of K_n). Further, every K_n is endowed with a (non-surjective) logarithm, i.e. with an embedding

$$\ell_n : (K_n^{>0}, \cdot, 1, <) \longrightarrow (K_n, +, 0, <)$$

such that ℓ_n extends ℓ_{n-1} and $\ell_n(K_n^{>0}) = K_{n-1}$. Then clearly, $\ell_\omega = \bigcup_{n\in\mathbb{N}} \ell_n$ will be a surjective logarithm on K_ω.

More precisely, if $\mathcal{U}_n^{>0}$ denotes the group of positive v-units of K_n , then the logarithm ℓ_n on $\mathcal{U}_n^{>0}$ will just be given by the logarithmic Taylor series. This makes K_n into a model of the theory of $\mathbb{R}$ with restricted exponentiation, cf. [4].

It then suffices to define the logarithm on a group complement $\mathbf{B}_n$ of $\mathcal{U}_n^{>0}$ in $(K_n^{>0}, \cdot, 1, <)$. To do so, we take a group complement $\mathbf{A}_n$ to the valuation ring R_n of K_n such that there is an embedding

$$h_n : G_n \longrightarrow \mathbf{A}_n$$

satisfying

$$vh_n(g) > g \quad \text{for all } g \in G^{<0} . \tag{1}$$

Then $h_\omega = \bigcup_{n\in\mathbb{N}} h_n$ will also satisfy (1). We then obtain ℓ_ω on $\mathbf{B}_\omega = \bigcup_{n\in\mathbb{N}} \mathbf{B}_n$ by composing h_ω with the canonical isomorphism $\mathbf{B}_\omega \Rightarrow G_\omega = \bigcup_{n\in\mathbb{N}} G_n$, $b \mapsto -v(b)$. It is then shown that condition (1) will guarantee that $f_\omega = \ell_\omega^{-1}$ is an exponential satisfying the axiom scheme

$$x > n^2 \implies f_\omega(x) > x^n \quad (n \in \mathbb{N}).$$

Hence by a theorem of Ressayre (cf. [3]), (K_ω, f_ω) will be a model of the theory of $(\mathbb{R}, \exp)$.

[1] F. V. KUHLMANN and S. KUHLMANN, *On the structure of nonarchimedean exponential fields II*, ***Communications in Algebra***, vol. 22 (1994), pp. 5079–5103.

[2] S. KUHLMANN, *On the structure of nonarchimedean exponential fields I*, to appear in ***Archive of Mathematical Logic***.

[3] J. P. RESSAYRE, *Integer parts of real closed exponential fields*, ***Arithmetic, proof theory and computational complexity*** (P. Clote and J. Krajicek, editors), Oxford University Press, New York, 1993.

[4] L. VAN DEN DRIES, A. MACINTYRE, and D. MARKER, *The elementary theory of restricted analytic functions with exponentiation*, to appear in ***Annals of Mathematics***.

[5] ———, *manuscript*.

► ANDREI A. KUZICHEV, *Equality of terms in the theory of C-monoids*.
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The theory of C-monoids (Cartesian closed categories with one object) can be interpreted in the $\lambda\beta\eta$-calculus with surjective pairing [1] (the calculus $\wedge_{\mathcal{S}}$). The usual analysis of equational theories of this kind is based upon the method of Church and Rosser. Unfortunately, many attempts to find Church - Rosser reductions, generating equality of terms in the theory of C-monoids or in $\wedge_{\mathcal{S}}$, fail (see the corresponding conjecture in [1]). However, we can analyze equality of terms in $\wedge_{\mathcal{S}}$, using the following generalization of the Church-Rosser property.

Let R be a reduction and K be a non-empty set of terms of some equational theory. We say that R is Church - Rosser over $K(R \in CR(K))$, if, for all terms a, b and c such that aRb, aRc and $b \in K$, there exists $d \in K$ such that bRd and cRd.

Let K be the set of all the terms of the pure λ-calculus. Let the reduction Red be the transitive closure of $R \cup Pr$, where R is some special Church-Rosser reduction, generating equality of terms in the $\lambda\beta\eta$-calculus, and where Pr is the reduction, defined by axioms of the forms $\pi_i < a_1, a_2 > Pra_i (i = 1, 2)$ and $\langle \pi_1 a, \pi_2 a\rangle Pra$.

THEOREM. $Red \in CR(K)$.

It means that $\wedge_{\mathcal{S}}$ is a conservative extension of the $\lambda\beta\eta$-calculus. As an application of this result, we get an elementary consistency proof for $\wedge_{\mathcal{S}}$ and for the theory of C-monoids.

[1] W. S. HATCHER and P. J. SCOTT, *Lambda-algebras and c-monoids*, ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 32 (1986), pp. 415–430.

► GUNWON LEE, *Genesis (history of logic)*.
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We pursue the GENESIS of the formal systems as in REPUBLIC(509B). A formal system can be generated as a model which can be realized as a model; a model of becoming; genesis. A formal system which is consistent, hence can have a model, had been said to be DYNAMIS in Timaeus(31B). A formal system, as it is, is a myth in Timaeus (27–29) MYTHOS.

As in Woo-Suk Park, a formal system can be safely named as a fictional myth, MUTHOS. The consistency of the formal system can have a model or be dynamic. However only when the formal system has been realized as in Kleene, it can be said to be the system of genesis in Timaeus. Therefore the genesis of Timaeus can be formulated as follows:

(1) A myth can be dynamic if and only if the myth is consistent.

(2) A consistent myth can be genesis if and only if the consistent myth is physically realizable.

A formal system as a being OUSIA as in Cornford is a Platonic Form. And as it is consistent, it can be dynamic. Usually the formal system in Plato as a Form is received as the consistent, therefore there are no doubt that all the Platonic Forms are dynamic. However the becoming, genesis has been examined as a separated concept in Cornford. And this separated concept of genesis can be said to be the realizable consistent models in (1) and (2) of the above.

However there are a lot of formal systems which can not be consistent, hence the system can be said to be dynamic or the Platonic Forms. In that sense, the logical systems can be named as a myth as in Park, but all of them are not the formal systems. Hence myth can be the Platonic Forms or a mere NOEMA which can be in the other world or nowhere in Parmenides. Only the Platonic Forms can be dynamic enough to be able to generate. The generated Platonic Forms can be named as GENESIS.

[1] F. M. CORNFORD, ***Plato's cosmology***, The Bobbs-Merrill Company, 1975.

[2] WOO-SUK PARK, *The fictional metalogic of the fictionalist philosophy of mathematics*, ***Philosophy***, Korean Philosophical Association, Seoul, 1993, Autumn, pp. 357–398.

► KERKKO LUOSTO, *Hierarchies of monadic generalized quantifiers*.
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This study can be seen as a part of a larger research programme aimed at finding relationships between syntactic properties of a quantifier Q and the expressive power of the logics $\mathcal{L}_{\omega\omega}(Q)$, $\mathcal{L}_{\infty\omega}(Q)$ and $\mathcal{L}^{\omega}_{\infty\omega}(Q)$ generated by that quantifier. To pick up a few examples, consider the finite, relational vocabulary τ_Q of a Lindström quantifier Q. Denote by n_R the arity of $R \in \tau_Q$. Then the *arity* of the quantifier Q is

$$\mathrm{ar}(Q) = \max\{n_R \mid R \in \tau_Q\}.$$

Hella (1989) showed that for every $\alpha \geq 0$, the Magidor–Malitz quantifier Q^{n+1}_α is not definable in the logic $\mathcal{L}_{\infty\omega}(\mathbf{Q}_n)$ where $\mathbf{Q}_n$ is the collection of all quantifiers of arity n, whence the quantifiers Q^n_α form a strictly increasing hierarchy in expressive power. Oversimplifying, this means that the increase in arity ($n = \mathrm{ar}(Q^n_\alpha)$) accounts for the increase in the expressive power. This line of thought can be pursued even further. The *pattern* of a quantifier Q is

$$p_Q : \omega \to \omega,\ p_Q(n) = |\{R \in \tau_Q \mid n_R = n\}|.$$

Hence, two quantifiers Q and Q' have the same pattern iff there is a renaming $\varrho : \tau_Q \to \tau_{Q'}$. Hella, Luosto and Väänänen (1994) give a linear order $\leq$ on patters such that if $p < p'$, then there exists a quantifier Q' with $p(Q') = p'$ which is not definable in $\mathcal{L}_{\omega\omega}(\mathbf{Q}^*_p)$ where $\mathbf{Q}^*_p = \{Q \mid p_Q = p\}$; this result holds especially in the realm of finite structures. What is lost when the hierarchy is refined is that whereas Hella's methods provide us with a back-and-forth characterization for the elementary equivalence of $\mathcal{L}_{\infty\omega}(\mathbf{Q}_n)$ (and $\mathcal{L}^{\omega}_{\infty\omega}(\mathbf{Q}_n)$), the latter result is purely existential in nature and is simply based on cardinality arguments.

Let us now restrict ourselves to monadic finite vocabularies and, for definiteness, to finite structures. In this case, the pattern may be identified with the *width* of Q, $\mathrm{wd}(Q) = |\tau_Q| = p_Q(1)$, and a combinatorial criterium can be given for non-definability of Q in the logic $\mathcal{L}_{\omega\omega}(\mathbf{Q}^*_p)$ for a monadic pattern p. What is crucial here is that monadic structures admit a simple classification: for such $\mathfrak{M}$, the unary relations $R^{\mathfrak{M}}$, $R \in \tau$, divide its universe in parts, the sizes of which determine $\mathfrak{M}$ up to isomorphism. Let $\bar{\kappa}_{\mathfrak{M}} \in \omega^l$ be the sequence of natural numbers obtained in this way, and let

$$\mathcal{R}(Q) = \{\bar{\kappa}_{\mathfrak{M}} \mid \mathfrak{M} \in K_Q,\ \mathfrak{M} \text{ is finite}\}$$

where K_Q is the defining class of the quantifier Q. Thus, the problems related to monadic quantifiers can be reduced to questions about relations on ω. This leads to the combinatorial concepts of the *rank* of the relation, $r(R)$, and its variant $r_T(R)$ which takes the sums of the components into account. The intuitive meaning of the equation $l = r(R)$ is that the relation

$R \subset \omega^n$, $n \geq l$, is essentially of the arity l.

The main results are: 1° If a monadic quantifier Q is definable in terms of monadic quantifiers of width l, then $r_T(\mathcal{R}_Q) \leq 2^l$. 2° There are relations R on ω of arbitrary high ranks $r_T(R)$. As an application, I show that the non-trivial resumptions of the Härtig quantifier are not definable by monadic quantifiers.

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Let (w) be the set of all partitions of w. We order the set in the following way: for $X, Y \in (w)$, $X \leq Y$ iff each piece of X is a union of pieces of Y. We consider finite modification of the ordering and write $X \leq_* Y$ iff all but finitely many pieces of X are unions of pieces of Y.

Having the lattice $((w), \leq_*)$ we reformulate the definitions from the classical van Douwen diagram and obtain two collections of dual coefficients. We 'draw' the diagrams and prove a number of facts concerning their cardinals. We describe implications of *MA* and its negations, and some consistency results.

► PAOLO MANCOSU, *Kant and Bolzano on proofs by contradiction as a watershed between philosophy and mathematics*.
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In [Mancosu 1991] I showed how widespread was the concern about proofs by contradiction among philosophers and mathematicians in the seventeenth century. Cavalieri, Descartes, Barrow, Guldin, Wallis and others argued for the superiority of ostensive proofs over proofs by contradiction. The latter types of proof were considered inferior epistemologically and heuristically. Epistemologically, since they do not proceed from causes to effects and thus do not show why a result holds but only that it holds. Heuristically, because most proofs by contradiction, for example all the results proved by exhaustion by Archimedes, fail to show by which method the result was obtained, and thus again do not give insight into the real reasons for the validity of the results. This background provides the context for the central theme of the paper, i.e. Kant's statements about proofs by contradiction in the section of the *Critique of Pure Reason* called "Transcendental Doctrine of the Method" and Bolzano's reaction to Kant's point of view. In "The discipline of pure reason in regard to its proofs" Kant compares mathematical and transcendental proofs, and sets down rules for the appropriate use of pure reason in its proofs. In particular the third rule reads : "The third rule peculiar to pure reason, in so far as it is to be subjected to a discipline in respect of transcendental proofs, is that its proofs must never be apagogical, but always ostensive. "As a result of the transcendental dialectic Kant bans the use of apagogical proofs from transcendental philosophy but allows it in mathematics. Indeed in mathematics it is not possible "mistakenly to substitute what is subjective in our representations for what is objective, that is, for the knowledge of that which is in the object". This is because mathematics proceeds by construction of concepts in intuition. That last part of the paper analyzes Bolzano's philosophy of mathematics with emphasis on his stand on proofs by contradiction. In particular, I will argue that Bolzano's proof, contained in the §530 of the *Theory of Science*, which shows that every indirect proof can be reduced to a direct one must be seen in the greater context of his opposition to Kant's philosophy of mathematics and to Kant's use of proofs by contradiction as a watershed between philosophy and mathematics.

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We prove that the problem: *given two Horn clauses $\mathcal{H}1$ and $\mathcal{H}2$ in the language of the first order logic without equality, decide if $\mathcal{H}1$ finitely implies $\mathcal{H}2$* is undecidable.

1. Motivation and Previous Results: Questions concerning decidability of clause implication and similar problems are motivated by artificial intelligence and automated deduction and have been an area of some interest recently. That is because redundancy elimination is believed to be an important issue in optimization of automated theorem provers or logic programs and clausal knowledge bases. The problem if a (general) clause $\mathcal{H}$ implies a clause $\mathcal{G}$ has been proved to be undecidable by M. Schmidt-Schauss in 1988 ([10]). The decidability of a clause implication problem for Horn clauses (i.e. whether a Horn clause $\mathcal{H}$ implies a Horn clause $\mathcal{G}$) remained open until 1992 ([9]). In [8] a particular Horn clause was constructed, which implies an undecidable set of Horn clauses.

Meanwhile, some strong sufficient conditions for decidability of Horn clause implication where given ([11, 3, 4, 5, 10, 7]).

The world described by a knowledge base is always a finite set of objects. So, we can claim, that the example of a natural and practical question is not the decidability of the problem: *does there exist* ***any*** *structure in which a Horn clause $\mathcal{H}$ is satisfied and a Horn clause $\mathcal{G}$ is not* but rather of the one: *does there exist a* ***finite*** *structure in which a Horn clause $\mathcal{H}$ is satisfied and a clause $\mathcal{G}$ is not*. If the last is not true, we say that $\mathcal{H}$ finitely implies $\mathcal{G}$.

There are some similarities between decidability problems of clause implication and decidability problems of dependencies implication in relational database theory (see [6]).

Many types of dependencies have been considered and some results concerning undecidability of finite implication have been proved (e.g. [1], [2]). What is important for us is that in the development of the theory finite and classical implication problems were considered simultaneously. It is not the case in the area of clause implication problems, where, up to our knowledge, finite implication have not been considered before.

2. The Results: Let $IMPL = \{<\mathcal{H},\mathcal{G}>: \ \mathcal{H}$ finitely implies $\mathcal{G}\}$ and let $RES = \{<\mathcal{H},\mathcal{G}>: \ \mathcal{G}$ can be derived from $\mathcal{H}$ by resolution$\}$. Since RES is a recursively enumerable set and, as it was proved in [9], not a recursive set, and since $IMPL$ is co-recursively enumerable set it is clear that resolution is not complete for finite implication in the sense, that it does not enumerate all elements of $IMPL$. We prove that $IMPL$ is undecidable. Moreover, we prove that RES and co-$IMPL$ are recursively inseparable.

[1] C. BEERI and M. Y. VARDI, *The implication problem for data dependencies*, ***Proceedings of the International Colloquium on Automata, Language and Programming 1981*** (Berlin), LNCS, no. 115, Springer.

[2] A. K. CHANDRA and M. Y. VARDI, *The implication problem for functional and inclusion dependencies is undecidable*, ***SIAM Journal of Computing***, vol. 14 (1985), no. 3, pp. 672–677.

[3] G. GOTTLOB and A. LEITSCH, ***Fast subsumption algorithms***, LNCS, no. 204-II, Springer, Berlin, Heidelberg, 1985.

[4] ———, *On the efficiency of subsumption algorithms*, ***Journal of the Association for Computing Machinery***, vol. 32 (1985), pp. 280–295.

[5] ———, *Deciding Horn clause implication problem by ordered semantic resolution*, ***Computational intelligence II*** (F. Gardin and G. Mauri, editors), 1990, pp. 19–26.

[6] P. C. KANELLAKIS, *Elements of relational database theory*, ***Handbook of theoretical computer science*** (J. van Leeuwen, editor), vol. B, 1990.

[7] A. LEITSCH, *Deciding Horn clauses by hyperresolution*, ***Proceedings of computer science***

logic 89, LNCS, no. 440, Springer, 1989, pp. 225–241.

[8] J. Marcinkowski, *A Horn clause that implies an undecidable set of Horn clauses*, ***Proceedings of 1993 annual conference of the European Association for Computer Science Logic***, LNCS no. 832, Springer, 1993, pp. 233–237.

[9] J. Marcinkowski and L. Pacholski, *Undecidability of the Horn clause implication problem*, ***Proceedings of the 33rd annual symposium on the foundations of computer science***, 1992, pp. 354–362.

[10] M. Schmidt-Schauss, *Implication of clauses is undecidable*, ***Theoretical Computer Science***, vol. 59 (1988), pp. 287–296.

[11] R. B. Stillman, *The concept of weak substitution in theorem-proving*, ***Journal of the Association for Computing Machinery***, vol. 20 (1973), pp. 648–667.

► SERGEJ MARDAEV, *Smallest-fixed-point theorem for modal logic S4*.

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Smallest-fixed-point theorem for **Int**, **Grz** and **GL** were announced in [1, 2]. The theorem for **Int** follows from [3] also. For **S4** we prove that for a formula with parameters $\Box q_1, \dots, \Box q_n$ the smallest fixed point is constructed by iteration.

Modal propositional formulas are formed in the usual way from variables , the constant 1 and connectives &, $\vee$, $\neg$, $\Box$, $\Diamond$.

Connective $\equiv$ is usual abbreviation.

We define

$$\begin{aligned} \varphi^0(q_1, \dots, q_n) &= 1, \\ \varphi^{n+1}(q_1, \dots, q_n) &= \varphi(\varphi^n(q_1, \dots, q_n), q_1, \dots, q_n). \end{aligned}$$

Theorem 1. *Let $\varphi(p, \Box q_1, \dots, \Box q_n)$ be a modal formula with parameters $\Box q_1, \dots, \Box q_n$ such that all occurrences of p are positive. There is a number m such that*

$$\vdash_{S4} \varphi^m(\Box q_1, \dots, \Box q_n) = \varphi^{m+1}(\Box q_1, \dots, \Box q_n).$$

The theorem has the obvious corollary for **Int**, which also follows from [3].

[1] S. I. Mardaev, *Smallest fixed point theorem for intuitionistic logic and Grzegorczyk logic*, ***Logic colloquium 93, abstracts*** (Keele), ASL, 1993.

[2] ———, *Smallest fixed point theorem for modal logic GL*, ***Third international conference on algebra, abstracts*** (Krasnoyarsk), 1993, p. 414.

[3] W. Ruitenburg, *On the period of sequences $(A^n(p))$ in intuitionistic propositional calculus*, ***The Journal of Symbolic Logic***, vol. 49 (1984), no. 3, pp. 892–899.

► TAMÁS MIHÁLYDEÁK, *Partiality and dynamics in first order logic and in type theoretical logic*.

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As it is well known there are many possibilities to treat partial feature of information in logic. Different types of partial logic try to model some types of (first of all) semantic partiality on object level (and not on meta level). In the background a new informal concept of meaning appears (for example a characteristic difference can be made between sentence and proposition). It has many consequences not only for the philosophy of logic in general but for the meanings of central notions of logic.

In dynamic semantics the general starting point is (as Groenendijk and Stokhof mentioned) “that the meaning of sentences resides in its information change potential.”

My paper deals with the following questions:

(1) Is there any room in a logical system for the notion of meaning of partial semantics and for "information change potential" ?
(2) What type of language and what type of semantics do we need ?
(3) Is there any real connection between two notions of meaning ? Is there any possibility to represent these notions in a correct semantic system ? Are there two different notions of meaning or can we define only one concept to represent these aspects of meaning.
(4) What does a partial dynamic semantic means for the central notions of logic ?
(5) Is there any real difference from theoretical point of view between first order logic of partial and dynamic semantics and type theoretical one ?
(6) Do we need some kind of intensional logic to connect partiality and dynamics ?

► ALEXANDER MITIN, *Undecidability of elementary theories of certain groups*.
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Let G_1 be the group of invertible and Lebesgue's measure preserving transformations of segment [0, 1]. We identify two transformations if they differ on a set having zero measure.

Let G_2 be the group of measure preserving transformations of the set of reals.

Let G_3 be the group of invertible transformations of the set of integers.

The groups G_1 and G_2 are studied in ergodic theory [2]. V. V. Ryzhikov stated a hypothesis that elementary theories of all G_1 and G_2 and G_3 groups are undecidable. We confirm the hypothesis. For any group G of the groups G_1, G_2 and G_3, we prove the following

THEOREM. *There is a translation (effective procedure) producing, for any given closed formula of arithmetic, a closed formula of the group language, which is true in G if and only if the given formula is true.*

Other examples of groups having undecidable elementary theories are presented in [1].

The author is sincerely grateful to V. V. Ryzhikov and N. K. Vereshchagin for fruitful discussions.

[1] YU. L. ERSHOV, I. A. LAVROV, A. D. TAIMANOV, and M. A. TAITSLIN, *Elementary theories*, ***Uspekhi Matematicheskikh Nauk***, vol. 20 (1965), pp. 37–108.

[2] P. R. HALMOS, ***Lectures on ergodic theory***, Tokyo, 1956.

► MALIKA MORE, *Rudimentary representations of spectra*.
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The spectrum of a first-order sentence is the set of cardinalities of its finite models. Let $\mathcal{S}$ be the class of all spectra and let $\mathcal{F}_k$ (resp. BIN) be the class of all spectra represented by a sentence of a language containing only predicates of arity k (resp. one binary predicate) (see Fagin [1]). Let RUD be the set of unary rudimentary relations. We consider rudimentary relations as defined by (polynomially) bounded formulas of the relational language of arithmetics $\mathrm{L}_{PA} \equiv \{\leq, +, .\}$ (see Harrow [2]). The following inclusions hold $\mathcal{F}_1 \subsetneqq \mathrm{RUD} \subseteq \mathrm{BIN} \subseteq \mathcal{F}_2 \subseteq \cdots \subseteq \mathcal{F}_k \subseteq \cdots \subseteq \mathcal{S}$, but it is unknown whether they are strict. Besides, RUD = BIN is equivalent to RUD = $\mathcal{S}$ and implies the collapsing of Stockmeyer's hierarchy at a level strictly greater than 1 (see Woods [4]).

Here, we construct rudimentary representations of the above classes of spectra (see More [3]): *for any spectrum $\mathcal{S}$ of $\mathcal{F}_k$, there is an integer $q \geq 1$ such that the range of $\mathcal{S}$ under the function $f_{q,k}(x) = 2^{q.x^k}$ is a rudimentary set*.

Let S be a spectrum represented by a sentence ϕ of the language $L \equiv \{R_0, \ldots, R_{q-1}\}$ consisting of q predicates of arity k. Let $G = \langle \mathcal{G}, \mathcal{L} \rangle$ be a finite L-structure of cardinality n.

In order to encode G into an integer, we consider in turn each k-tuple of elements of $\mathcal{G}$, and we describe its structure concerning the q relations of L. So, the encoding of G is represented by a binary word c of length $q.n^k$ i.e. by an integer strictly smaller than $f_{q,k}(n)$.

Then we change the formulas f of L into bounded formulas of L_{PA} (denoted by $U(\phi)$) containing two new free variables n and c as follows: each atomic formula is changed into a bounded L_{PA} formula and each quantifier is bounded by n. Then the following property implies our result: $x \in S(\phi)$ if and only if $f_{q,k}(x) = 2^{q.x^k}$ satisfies $\exists n \leq m \exists c \leq m(m = f_{q,k}(n) \wedge U(\phi)(n,c))$.

Any function g, rudimentary as a relation and asymptotically greater than the function $f_{q,k}$ asymptotically changes any spectrum represented by a sentence of L into a rudimentary set. For instance, the function defined by $f(x) = 2^{x^x}$ (resp. $f_k(x) = 2^{(x^{2k})}$), which is asymptotically greater than all functions $f_{q,k}$ (resp. with fixed k) allows us to represent all spectra (resp. all spectra of $\mathcal{F}_k$) as rudimentary sets, provided that some cases are dealt with by hand.

[1] R. Fagin, *A spectrum hierarchy*, ***Zeitschrift für Mathematiche Logik und Grundlagen der Mathematik***, vol. 21 (1975), pp. 123–134.

[2] K. Harrow, *Sub-elementary classes of functions and relations*, ***Ph.D. thesis***, New York University, 1973.

[3] M. More, *Définissabilité dans les graphes finis et en arithmétique bornée*, ***Ph.D. thesis***, Université d'Auvergne, Clermont, 1994.

[4] A. Woods, *Some problems in logic and number theory, and their connections*, ***Ph.D. thesis***, University of Manchester, 1981.

► A. S. MOROZOV, *Automorphisms of decidable models*.
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We study the interplay between decidability for models and their automorphism groups. The results obtained imply that there is no specific relations between decidability of models and isomorphism types of automorphism groups of these, but if we consider such groups as sets of transformations then the situation is changed.

All necessary definitions may be found in [1].

A model M is said to be *recursive (decidable)* provided its universe is a recursive subset of ω and one can effectively recognize quantifier–free (all first–order) formulas $\varphi(\bar{x})$ and n–tuples $\bar{m}$ such that $M \models \varphi(\bar{m})$. $\mathrm{Aut}_r M$ stands for the recursive automorphism group of M.

Theorem 1. *There exists a recursive model M such that for all decidable models N of arbitrary effective language Aut $M \neq$ Aut M and $Aut_r M \neq Aut_r N$ hold.*

Theorem 2. *For each recursive model M there exists a decidable model N such that*

$$Aut\ M \cong Aut\ N.$$

Theorem 3. *There exists a decidable ω–categorical model M such that for each recursive model N*

$$Aut_r N \cong Aut_r M^\star$$

holds for some decidable model $M^\star \cong M$.

Considering constructive models, we can reformulate this result as follows:

Corollary. *There exists a ω–categorical countable model M such that the class of recursive* *automorphism groups coincides with*

$$\{\ Aut_r(M, v) \mid v \text{ is a strong constructivization of } M\}$$

up to the isomorphism.

[1] YU. L. ERSHOV, ***Decidability problems and constructive models***, Nauka, Moscow, 1980.

► LUDOMIR NEWELSKI, *Trace calculations and local $\mathcal{M}$-ranks*.
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Assume T is superstable and small. For a type $s(x)$ and a set A let the trace of s on A be

$$Tr_A(s(x)) = \{stp(a/A) : s(a) \text{ holds}\}$$

. We show some basic properties of this notion, similar e.g. to the open mapping theorem of Lascar-Poizat.

I defined a multiplicity rank $\mathcal{M}$ on complete types over finite sets by the following condition:

(a) $\mathcal{M}(a/A) \geq \alpha + 1$ iff $\exists$ finite $B\big[B \supseteq A,\ a \smile\!\!\!\!|\ B(A),\ \mathcal{M}(a/B) \geq \alpha$ and $Tr_A(a/B)$ is nowhere dense in $Tr_A(a/A)\big]$.

This rank yields meager (hence locally modular, regular) types (many of them if $I(T, \aleph_0) < 2^{\aleph_0}$). This is similar to U-rank yielding regular types. Also, $I(T, \aleph_0) < 2^{\aleph_0}$ implies $(\mathcal{M}(p) \geq \omega^\beta \implies R_\infty(p) \geq 1 + 2\beta)$.

It is open whether $I(T, \aleph_0) < 2^{\aleph_0}$ implies $\mathcal{M} \leq U$.

Now fix a finite set C. We define local ranks $\mathcal{M}_C$ and $\mathcal{M}_C^e$ on types $p \in S(A)$, $A \supseteq C$ finite, by the following conditions:

(b) $\mathcal{M}_C(a/A) \geq \alpha + 1$ iff $\exists$ finite $B\big[B \supseteq A, a \smile\!\!\!\!|\ B(A), \mathcal{M}_C(a/B) \geq \alpha$ and $Tr_C(a/B)$ is nowhere dense in $Tr_C(a/A)\big]$.

(c) $\mathcal{M}_C^e(a/A) \geq \alpha + 1$ iff $\exists$ finite $B, D\big[A \subseteq B \subseteq D, a \smile\!\!\!\!|\ D(A), \mathcal{M}_C^e(a/D) \geq \alpha$ and $Tr_C(aB/D)$ is nowhere dense in $Tr_C(aB/B)\big]$.

$\mathcal{M}_C^e$ is an eventual version of $\mathcal{M}_C$ and has the additivity properties of $\mathcal{M}$ (while $\mathcal{M}_C$ does not). $\mathcal{M}_C^e$ also yields meager types, related more closely to C.

► A. I. OMAROV, *The syntactical description of filtered formulas*.
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A formula $\varphi(x)$ is *a filtered formula*, if for every filter $\langle J, E\rangle$, every collection of similar algebraic structures $\mathfrak{M}_i$, $i \in J$, and every $\bar{f} \in \prod \mathfrak{M}_{i/E}$ the following is satisfied:

$$\prod \mathfrak{M}_{i/E} \models \varphi(\bar{f}) \iff \{i \mid \mathfrak{M}_i \models \varphi(\bar{f}(i))\} \in E.$$

We give a syntactical description of filtered formulas. E. A. Palyutin introduced a new class of formulas, called by P-formulas. The P-formulas are built by induction:

(1) atomic formulas are P-formulas;
(2) the set of P-formulas is closed under conjunction;
(3) if φ and ψ are P-formulas, then $(\exists x)\varphi \& (\forall x)(\varphi \longrightarrow \psi)$ is a P-formula.

THEOREM 1. *Let the Boolean algebra $2^J/E$ contains a nontrivial atomless element. Then a formula $\varphi(X)$ is filtered by the filter $\langle J, E\rangle$ iff it is equivalent to some P-formula.*

There are three different classes of filtered formulas:

(1) filtered by ultrafilter; by Łoś' Theorem this is the class of all formulas;
(2) filtered by filter $\langle J, E\rangle$ such that $2^J/E$ is an atomic Boolean algebra.
(3) filtered by filter $\langle J, E\rangle$ such that $2^J/E$ has an atomless element ; it is exactly the class of P-formulas

The class (b) of formulas is called *multiplicative*.

We give some theorems about multiplicative formulas as analogies to the corresponding theorems about P-formulas.

[1] A.I. Omarov, *On the multiplicative formulas*, ***Algebra i Logika***, vol. 32 (1993), no. 2, pp. 139–147, (in Russian).

► DMITRY E. PAL'CHUNOV, *Relatively finitely axiomatizable boolean algebras with distinguished ideals*.
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Boolean algebras with $\lambda \in \omega$ distinguished ideals (called below as I-algebras) are studied in the signature $\langle \cup, \cap, , 0, 1, I_1, \ldots, I_\lambda\rangle$. All definitions one can find in [1].

Denote by $\mathfrak{L}^\lambda$ the Lindenbaum-Tarski algebra of the I-algebra class, and by F_n the n-th Frechet ideal of $\mathfrak{L}^\lambda$. Denote $T_n \rightleftharpoons \{\varphi \in \mathfrak{L}^\lambda \mid \neg\varphi \in F_n\}$. An I-algebra $\mathfrak{A}$ is called non-vanishing if for any decomposition $\mathfrak{A} = \mathfrak{B} \times \mathfrak{C}$, we have $\mathfrak{A} \equiv \mathfrak{B}$ or $\mathfrak{A} \equiv \mathfrak{C}$. A finitely axiomatizable non-vanishing I-algebra is called basic one. If $\mathfrak{A} \equiv \mathfrak{B} \times \mathfrak{C}$, then we denote $\mathfrak{B} \leq \mathfrak{A}$; $\mathfrak{B} < \mathfrak{A}$ means that $\mathfrak{B} \leq \mathfrak{A}$ and $\mathfrak{B} \not\equiv \mathfrak{A}$. A non-local I-algebra $\mathfrak{A}$ is called minimal if $\mathfrak{B} \equiv \mathfrak{A}$ for any non-local $\mathfrak{B} \leq \mathfrak{A}$.

Definition. An I-algebra $\mathfrak{A}$ is relatively finitely axiomatizable if for some $n \in \omega$ the I-algebra $\mathfrak{A}$ is finitely axiomatizable in the theory T_n and $\mathfrak{A} \models T_n$.

Theorem 1. *An I-algebra $\mathfrak{A}$ is called relatively finitely axiomatizable iff the number of non-local non-vanishing direct summands of $\mathfrak{A}$ is finite and for any minimal non local $\mathfrak{B} \leq \mathfrak{A}$ the following statements hold.*

a) *Almost for all basic $\mathfrak{C} \leq \mathfrak{B}$ there exists only (modulo the elementary equivalence) basic $\mathfrak{D} \leq \mathfrak{B}$ such that $\mathfrak{D} \not\leq \mathfrak{C}$ and $\mathfrak{C} \not\leq \mathfrak{D}$.*

b) *If $\mathfrak{B} < \mathfrak{C} \leq \mathfrak{A}$ and $\mathfrak{C}$ is non-vanishing, then $\mathfrak{C}^n \not\leq \mathfrak{A}$ for some $n \in \omega$.*

c) *There exists local $\mathfrak{C} \leq \mathfrak{A}$ such that for any basic $\mathfrak{D} \leq \mathfrak{A}$ if $\mathfrak{D} \not\leq \mathfrak{C}$, then $\mathfrak{D} \leq \mathfrak{M}$ for some minimal non-local $\mathfrak{M} \leq \mathfrak{A}$.*

Theorem 2. *For any relatively finitely axiomatizable I-algebra $\mathfrak{A}$, the elementary theory $Th\,\mathfrak{A}$ is decidable.*

Theorem 3. *For any $n \in \omega$, the sentence set F_n and the theory T_n are decidable.*

[1] D. E. Pal'chunov, *Direct summands of boolean algebra with distinguished ideals*, ***Algebra and logic***, vol. 31 (1992), no. 5, pp. 499–537.

► LORENZO PEÑA, *Strengthening entailment logic into a logic of fuzziness*.
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We propose a chain of strengthenings of system **E**. The **E** functor '→', duly strengthened, receives a construal as meaning 'to the extent [at least] that'. The resulting systems are closer to **CL** (Classical Logic). They are not relevant, in so far as they fail to comply with the relevantist constraint about variable-sharing; yet, they comply with the constraint of use in proof, with some adaptations. The resulting systems of fuzzy or gradualistic logic are intermediary between classical and entailmental logic. The approximation to **CL** goes further than that. In fact, several of the systems contain the whole **CL**, and are indeed conservative extensions thereof. Only, the **CL** negation '¬' is given a different reading from what is customary. It is now read 'not...at all'. Thus viewed, classical negation is complete negation - the functor which maps anything true, to whatever extent, into complete falsehood, and the utterly false into complete truth. In some important sense, those systems are to **CL** as relevant logic was meant to be to intuitionistic logic. All those systems have the laws of non-contradiction and excluded middle. They are paraconsistent copulative systems.

- P0 = E P1 = P0 + Factor
- P2 = P1 + Linearization ($\ulcorner p \to q \vee q \to p \urcorner$)
- P3 = P2 + the Self-Self Principle ($\ulcorner N(p \to p \to N(p \to p)) \to .p \to p \urcorner$)
- P4 = P3 + **IF** [Implicational Funnel] ($\ulcorner p \to q \vee .p \to q \to r \urcorner$)
- P4.5 = P4 + Mingle = **CL**. Mingle is $\ulcorner p \to .p \to p \urcorner$
- P5 = P4 + one of these: Aristotle ($\ulcorner N(p \to Np) \urcorner$), Boethius ($\ulcorner p \to q \to N(p \to Nq) \urcorner$) or Contradiction (namely, for some particular sentential constant j: $\ulcorner j \wedge Nj \urcorner$)
- P6 = P4 + both these principles: $\ulcorner p \to q \to .NHNHp \to Hq \urcorner$ and $\ulcorner Hp \to q \vee .Np \to r \urcorner$ ('H' is read: 'It is altogether true that')
- P3.5 = P3 + those two principles involving 'H' = P6 minus **IF**
- P7 = P6 + $\ulcorner Hp \vee Np \urcorner$
- P8 = P3.5 + these two: $\ulcorner \alpha \urcorner$ and $\ulcorner \alpha \to p \vee .p \to q \urcorner$ (' α ' is a sentential constant meaning the conjunction of all truths)
- P8.5 = P8 + $\ulcorner H\alpha \urcorner$ = **CL** P9 = P8 + $\ulcorner NH(\alpha \to \alpha) \urcorner$ P10 = P8 + $\ulcorner HN(N\alpha \to \alpha) \urcorner$

P8 contains P7; P10 contains P9; P9 contains P5; P8 is a conservative extension of CL (if classical negation, '¬', is defined as 'HN'); P5 and such systems as they contain P5 are not just paraconsistent but contradictorial (they have the theorem of Heracleitus: $\ulcorner N(p \to p) \urcorner$). (As is well-known, $\ulcorner N(p \to p) \to (p \to p) \to (p \to p) \to N(p \to p) \to N(N(p \to p) \to (p \to p) \to (p \to p) \to N(p \to p)) \urcorner$ is a theorem of **E**; the apodosis is the negation of the protasis; when we add Aristotle or Boethius, the negation of the theorem becomes a theorem, too. But in P1 we prove $\ulcorner p \wedge Np \to N(q \to q) \urcorner$). System P8 and those above it allow to implement classical conditional (which can be defined in the usual way with strong negation, '¬', and disjunction, '∨') in the relevantly recommended way: $p \supset q$ iff there is some truth, namely α, such that $p \wedge \alpha \to q$.

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Theories T and S are said to be *semantic similar over the list L of model-theoretic properties*, if there is a recursive isomorphism $\mu : L(T) \longrightarrow L(S)$ between their Lindenbaum algebras such that any complete extension $T*$ of the theory T and the corresponding complete extension $S*$ of theory S, $S*$ and $\mu(T*)$ have identical description over properties of L.

Let σ be a finite signature, and $SL(\sigma$ be the set of all sentences of σ considered as finitely axiomatizable theories. The class $E \subseteq SL(\sigma)$ is a *semantic class over* L if E is closed under semantic similarity over the list L. Relations of semantic similarity appeared in connection with expressiveness problem of finitely axiomatizable theories [1]. Semantic classes are natural analogues to index set in algorithm theory. Some analogues of Rice's theorem (levels up to 3) are proved for this classes. We investigate some other properties of semantic classes.

A semantic class E is called *weakly decidable* if all theories F of E are decidable, and is called *uniformly decidable* if there is an algorithm representing the uniform procedure to the decision problem for all theories of E.

THEOREM 1. *Let E be a semantic class over the list L only consisting of complete theories. Then E is uniformly decidable.*

THEOREM 2. *Let E be a semantic class over the list $L = 0$ which is weakly decidable. Then E is uniformly decidable if and only if E only consists of complete theories.*

[1] M. G. PERETYAT'KIN, *Expressive power of finitely axiomatizable theories, parts I, II, III*, ***Siberian Advances in Mathematics***, vol. 3 (1993), no. 2, pp. 153–197, no. 3, pp. 123–145, no. 4, pp. 131–201.

► REGIMANTAS PLIUŠKEVIČIUS, *Saturated disjunction property for a linear temporal logic*.
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We consider a propositional linear temporal logic with $\bigcirc$ ("next") and $\Box$ ("always") as temporal operators. The sequent S is called primary if

$$S = \Sigma_1, \bigcirc^n \Pi_1, \bigcirc^n \Box\Omega \to \Sigma_2, \bigcirc^n \Pi_2, \bigcirc^n \Box\Delta \ (n > 0), \text{ where } \Sigma_1, \Sigma_2, \Pi_1, \Pi_2 = \varnothing$$

consists of logical formulas without $\bigcirc$ and $\Box$; $\Omega, \Delta = \varnothing$ or consists of logical formulas without $\bigcirc$.

LEMMA 1. *$\vDash (S \iff \bigwedge_{i=1}^{n} S_i)$, where $S_i (1 \leqslant i \leqslant n)$ is a primary sequent; the set $\{S_1, \dots, S_n\}$ is called a primary expansion of S and denoted by $P(S)$.*

LEMMA 2. *Let $S \in P(S)$, then $\vDash S \Rightarrow \vDash \Sigma_1 \to \Sigma_2$ or $\vDash S \Rightarrow S^* = \Pi_1, \Box\Omega \to \Pi_2, \Box\Delta$; the sequent S^* is called the alternating one.*

Let us define the notion of the resolvent of a sequent S. Let us apply Lemma 2 to S_j from $P(S)$ and let $S^*_{j_2}$ be the alternating sequent of this application.

Let $R^*e(S) = \{S^*_{1_2}, \dots, S^*_{n_2}\} = \{S_1, \dots, S_k\} (k \leqslant n)$. Let us drop from $R^*e(S)$ all members which can be obtained from other by the structural rule of weakening. Then the obtained set will be called a resolvent of S and denoted by $Re(S)$.

Let us define the notion of the k-th resolvent of a sequent S (denoted by $Re^k(S)$) and the notion of the saturated sequent. $Re^0(S) = S$; $Re^1(S) = Re(S)$.

Let $S_i \in Re^k(S)$ and $R^*e^{k+1}(S) = \bigcup_i Re(S_i)$. Then $Re^{k+1}(S)$ is obtained from $R^*e^{k+1}(S)$ by dropping all the sequents that can be obtained from some sequent $S^* \in R^*e^{k+1}(S)$ or by some sequent $S' \in Re^l(S) (0 \leqslant l < k+1)$ by the structural rule of weakening. Let for some n $Re^n(S) = \varnothing$, then the set $\bigcup_0^n Re^{n-1}(S)$ will be called the saturated one of S and denoted by $\mathrm{Sat}\{S\}$.

LEMMA 3. *Let $S^* \in Sat\{S\}$, then $Re(S^*) \in Sat\{S\}$ or can be obtained from some $S' \in Sat\{S\}$ by the structural rule of weakening.*

THEOREM 1. *Let $\vDash S = \Pi_1, \Box\Omega \to \Pi_2, \Box A_1, \dots, \Box A_n$ and S be saturated, then there exists $\Box A_j$ such that $\vDash \Pi_1, \Box\Omega \to \Pi_2, \Box A_j$ or $\vDash \Box\Omega \to \Box A_1, \dots, \Box A_n$ $(1 \leqslant j \leqslant n)$.*

REMARK. Let $S = P, \Box(P \supset (\bigcirc P \wedge \Box(C \vee D) \wedge \Box(C \supset \bigcirc C) \wedge \Box(D \supset \bigcirc D))) \to \Box C, \Box D$, then $\vDash S$, however S is not saturated and Theorem 1 fails for the sequent S.

Theorem 1 is the main point for the generalization of [1], [2] to the case of non-Horn-like sequents.

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[2] ———, *On saturated calculi for a linear temporal logic*, Lecture Notes in Computer Science, no. 711, Springer, 1993, pp. 640–649.

► JORDI REBAGLIATO AND VENTURA VERDÚ, *The deduction theorem for Gentzen systems*.
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Let $\mathcal{L}$ be a propositional language and let $\mathcal{G}$ be an n-dimensional Gentzen system (see the abstract *Algebraizable Gentzen systems*.

We define the notion of the deduction-detachment theorem for an n-dimensional Gentzen systems $\mathcal{G}$. When $\mathcal{G}$ is an n-dimensional deductive system we obtain the deduction detachment theorem stated in [3] and [2].

Let $\mathcal{G}$ be an algebraizable Gentzen system and let K be its equivalent quasivariety semantics. We prove that $\mathcal{G}$ has the deduction-detachment theorem iff the quasivariety K has equationally definable principal relative congruences. The proof of this theorem is similar to the proof of the analogous theorem for deductive systems given in [2].

Several examples are given.

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[2] ———, *The deduction theorem in algebraic logic*, Preprint, 1989.

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[4] J. Rebagliato and V. Verdú, *El teorema de la deducción para sistemas de Gentzen*, ***Actas del I Congreso de la Sociedad de Lógica, Metodología y Filosofía de la Ciencia*** (Madrid) (E. Bustos, J. Etchevería, E. Pérez Sede no, and M. I. Sánchez Balmaseda, editors), 1993, pp. 118–120.

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[7] G. Takeuti, ***Proof theory***, Studies in Logic, no. 81, North-Holland, 1975.

► VLADIMIR V. RYBAKOV, *Tabular modal and intermediate logics without independent basis for admissible inference rules*.
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Modal logic has nowadays a lot of applications to Computer Science. Tabular modal logics (logics with the set of truth values forming a finite algebra) have some special interest because finite many-valued logics reflect behavior of programs carrying out computations. Inference rules, generally speaking, simulate programming instructions. The study of inference rules for tabular (finite many-valued) logics is rather developed popular field. In [2] W. Rautenberg proved that every 2-element matrix has finite base for valid inference rules. P. Wojtylak [4] constructed 5-element matrix which has no finite basics for valid inference rules, 4-element matrix with this property was found later by W. Dziobiak [1]. Similar results are known for tabular modal logics. In [3] an example of tabular modal logic without finite basis for valid inference rules was found and it is show that any finite subdirectly irreducible modal K4-algebra has a finite base for valid inference rules. A considerably old open problem is :

whether any tabular modal logic has finite base for admissible inference rules (all those with respect to which the logic is closed) ? Note that admissible rules contain among them all valid inference rules but not conversely. We are going to answer this question.

There exists a finite generated (rooted) partially ordered set (frame) **F** of depth 3 such that

THEOREM 1. *The tabular modal logic* **LM(F)** *of the frame* **F** *has no basis for admissible rules in finitely many variables (in particular, has no finite basis). The superintuitionistic tabular intermediate logic* **LI(F)** *generated by* **F** *has no basis in finitely many variables for admissible inference rules.*

THEOREM 2. *The quasi-varieties generated by free algebras of countable rank from varieties of modal and pseudoboolean algebras corresponding to* **LM(F)** *and* **LI(F)** *have in these varieties only finite number of covering quasi-varieties.*

From this theorems we immediately extract:

THEOREM 3. *Tabular logics* **LM(F)** *and* **LI(F)** *do not have even independent basis for admissible inferences rules.*

In the same time, logics **LM(F), LI(F)** have some finite basis for valid inference rules by [3].

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[2] W. RAUTENBERG, *2-element matrices*, ***Studia Logica***, vol. 40 (1981), pp. 315–353.

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[4] P. WOJTYLAK, *An example of a finite through finitely non-axiomatizable matrix*, ***Reports on Mathematical Logic***, vol. 17 (1984), pp. 39–46.

► ANDREAS SCHLÜTER, *A theory of rules for enumerated classes of functions*.
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We define an applicative theory $\mathbf{CL}_2$ similar to combinatory logic which can be interpreted in classes of functions possessing an enumerating function in the same way as classical combinatory logic is interpreted in the recursion-theoretic (partial) model. In contrast to the classical case, it is not necessarily assumed that the enumerating function itself belongs to that function class. Thereby we get a variety of possible models including e.g. the classes of primitive recursive, recursive, partial recursive, elementary, polynomial-time computable or ε_0-recursive functions.

We show that in $\mathbf{CL}_2$ a major part of the metatheory of enumerated classes of functions can be developed. Namely, a kind of λ-abstraction can be defined and abstract versions of the S_n^m- and (primitive) recursion theorems are proved.

A theory closely related to $\mathbf{CL}_2$ can be used to replace the applicative part of Feferman's theories for explicit mathematics. So this work can also be seen as an answer to a question by Feferman to formulate a theory for explicit mathematics in which operations can be interpreted as primitive recursive or even more feasible ones.

Finally, the proof-theoretical strength of theories with primitive recursive operations is compared to the classical systems of explicit mathematics formalizing general recursive operations.

[1] ANDREAS SCHLÜTER, *A theory of rules for enumerated classes of functions*, to appear in ***Archive for Mathematical Logic***.

► ANTON SETZER, *Proof theoretical strength of Martin-Löf type theory*.
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THEOREM. *The proof theoretical strength of extensional and intensional Martin-Löf Type Theory with W-type and one universe in the formalization* à la Russel *is* $\psi_{\Omega_1}(\Omega_{I+\omega})$.

The proof of this theorem has two directions, here we will concentrate on the proof of an upper bound by embedding Martin-Löf Type Theory in a set theory other versions as well. KPi^+ is an extension of Kripke-Platek set theory by adding axioms demanding the existence of a recursive inaccessible I and of finitely many admissibles above it.

We will essentially interpret closed types as the the set of terms, introduced by some introductory rule, and close this set under reduction. For the interpretation of the W-type we have an inductive definition, which we carry through by iterating an operator up to the next admissible. In the case of the universe we need a fixed point of an operator, which leads to the next admissible (since the universe is closed under W-type), therefore we iterate an operator up to an admissible, closed under admissibles, namely I. In order to interpret W-types, which are built using the universe, we need finitely many admissibles above I.

We conclude, that all Π^1_1-sentences provable in Martin-Löf Type Theory are provable in KPi^+ (we only have to restrict quantification over sets of natural numbers to elements of the least admissible containing ω). Therefore Tarski and Russel version of the Type Theory have as upperbound the strength $\psi_{\Omega_1}(\Omega_{I+\omega})$, which is "slightly" bigger than the strength of $(\Delta^1_2 - CA) + BI$, KPi and Feferman's theory T_0.

For the proof of the lower bound, which can be found in [2], we can only give a flavour. We use essentially, that we can achieve W-types with branching degrees, which are not elements of the universe, by using the sum of products of types, where the first component is possibly empty depending on the boolean value of a primitive recursive relation. Using this and well ordering techniques from proof theory we can show, that Martin-Löf Type Theory proves transfinite induction up to $\psi_{\Omega_1}(\Omega_{I+n})$.

[1] JAEGER, ***Theories for admissible sets: A unifying approach to proof theory***, Bibliopolis, 1986.

[2] A. SETZER, *Proof theoretical strength of Martin-Löf type theory with W-type and one universe*, ***Ph.D. Thesis***, Universität München, 1993.

► ALTYNBEK SHARIPBAY, *Construction of proof theory for digital circuits*.
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The proof theory on language of unary predicates of the second order is constructed for Digital Circuits (DC), which process some input signals, coming in discrete moments of time, into output signals.

In this language there are one constant "1" initial moment of time and one unary functional symbol "$\uparrow$" immediate successor, two kinds of variables: $t_1, t_2, \dots, t_s$—the moments of time and $X_1, X_2, \dots, X_k$—the string of signals in given set of their states. Terms are defined as usually. Atomic formulas is as like as $X(t) = \alpha$, which means that at th t-th moment of time the state of signal X is α, where "=" is graphical equality. Formulas are produced from atomic by the application of logic operations and introducing quantifiers to the both kinds of variables. There are the rules of deduction for admissible connections of DC's.

Behavioural and structural specifications for the hierarchy of gradually complicated DC's are constructed by those formulas and rules: the names of behavioural specifications of basic DC considered as axioms, provide the first level, then applying rules of deduction to these axioms we can construct structural specifications for the second level of hierarchy and we can use the behavioural specifications of the second level as axioms to construct structural specifications for third level etc.

There is the theorem about completeness of this theory. The problem of verification for DC is algorithmically decidable.

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Neighbourhood (or topological) semantics for modal and intermediate logics dates back to Tarski and Tang (1938). It is well-known that every Kripke-complete logic is neighbourhood-complete. The converse holds neither for S4-logics , nor for intermediate logics ([2, 4]). On the other hand, there exist neighbourhood-incomplete S4-logics [3]; for intermediate logics, the corresponding problem remains open.

A related notion is "strong completeness" (or "compactness"): a logic L is called strongly Kripke-complete (S-K-complete) if every L- consistent theory has a model on some frame for L. The corresponding definition has sense also in the neighbourhood semantics (S-N-completeness).

DEFINITION. A class of generated Kripke S4-frames is called "steady" if it is closed under generated subframes and adding roots. A modal logic is called "steady" if it is complete w.r.t. a steady class.

THEOREM 1. *Every steady S4-logic is S-N-complete.*

THEOREM 2. *Every Kripke-complete intermediate logic is S-N-complete.*

Due to some results from [1] and [5], these theorems allow to construct many examples of S-N-complete logics which are not S-K-complete.

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[2] M. GERSON, *?*, ***Studia Logica***, vol. 34 (1975), pp. 334–342.
[3] ———, ***The Journal of Symbolic Logic***, vol. 40 (1975), pp. 141–147.
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[5] T. SHIMURA, *On strong Kripke completeness of intermediate logics*, Manuscript, 1992.

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Joint work with Ehud Hrushovski.

We will indicate how to obtain a strong structure theory for strongly minimal sets definable in a differentially closed fields of characteristic zero. It turns out that strongly minimal sets are locally modular, or else equivalent to the field of constants. Using some classical objects one explicitly describes the locally modular nontrivial strongly minimal sets up to nonorthogonality, and as a consequence shows that there are continuum many nonisomorphic countable differentially closed fields proving a conjecture of S. Shelah.

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We want to generalize the Grzegorczyk hierarchy $\{E_n\}_{n\geq 0}$ on natural numbers and

Weihrauch's Grzegorczyk hierarchy $\{E_n^k\}_{n\geq 0}$ on words over an alphabet of size $k \geq 2$ to Grzegorczyk hierarchies on arbitrary term algebras. By a term algebra $\mathcal{T}^\sigma$ of signature σ we mean the set of terms generated by a set of finitely many function symbols with at least one of arity 0 and ≥ 2.

As there is a canonical notion of primitive recursion for functions on term algebras, we have the hierarchy of recursion number classes $\{R_n^\sigma\}_{n\geq 0}$ on term algebras.

Terms can be considered both as words and trees; the measures of length and height of a term induce quasi orderings on $\mathcal{T}^\sigma$, which are appropriate to define two notions of bounded primitive recursion. After introducing two different sequences of Ackermann functions, the corresponding Grzegorczyk hierarchies $\{L_n^\sigma\}_{n\geq 1}$ and $\{H_n^\sigma\}_{n\geq 0}$ are defined. Both hierarchies coincide for all (super)elementary classes (i.e. for $n \geq 3$), and even the well-known theorem "$R_n = E_{n+1}$ for all $n \geq 2$" can be transferred to $\{L_n^\sigma\}$ and $\{H_n^\sigma\}$. However they are incomparable in the subelementary cases. $\{E_n\}$ and $\{E_n^k\}$ can trivially be embedded in $\{L_n^\sigma\}$ and $\{H_n^\sigma\}$ for a suitable σ. But which of these hierarchies is the "best generalization" of $\{E_n\}$ and $\{E_n^k\}$?

On the one hand $\{H_n^\sigma\}$ is related to $\{R_n^\sigma\}$ like $\{E_n^{(k)}\}$ is to $\{R_n^{(k)}\}$, while $\{L_n^\sigma\}$ is not for $n = 1, 2$. But some important functions, e.g., *length*(t), *equal*(s,t), are not resp. not obviously in H_2^σ. For that reason $\{H_n^\sigma\}$ doesn't provide a good notion of feasibility.

On the other hand, for each $n \geq 2$ the class L_n^σ turns out to be computationally equivalent to a word class E_n^k, and hence to a Turing machine time computation class. Thereby several natural functions like *length*(t), *height*(t), *equal*(s,t), *treeunion*(s,t) can easily be shown to be efficiently computable. Especially L_2^σ is equivalent to polynomial time computability. So there exists a notion of feasible term algebra functions. Some properties of $\{E_n\}$ and $\{E_n^k\}$ carry over to $\{L_n^\sigma\}$; for example: L_{n+1}^σ is universal for L_n^σ for all $n \geq 2$.

In this sense $\{L_n^\sigma\}$ is the better generalization of $\{E_n\}$ and $\{E_n^k\}$.

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Properties of monotonic inference systems and properties of their theories are strongly linked. These links, however, are much weaker in nonmonotonic inference systems. In the absence of monotonicity, the meaning of a theory, as well as the role of theories in the analysis of inferences is radically different from the role theories play in monotonic calculi. Theories alone are not sufficiently expressive for the analysis of nonmonotonic inferences.

In the context of nonmonotonicity another syntactic notion assumes the role a theory plays in monotonic inference systems. The notion of the axiomatic variety for a theory captures all possible ways a theory can be axiomatized. With this notion, we recover many syntactic and semantic links that exist between monotonic inference systems and their theories but which are absent in nonmonotonic context. In this paper we introduce axiomatic varieties and study their basic properties. We show how axiomatic varieties can be used to identify, define, and compare nonmonotonic inference systems. The semantic links between properties of axiomatic varieties and properties of inference systems are also discussed. We formulate syntactic and semantic criteria for selected properties of nonmonotonic calculi (cummulativity, loop-cummulativity, and cut).

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Since the discovery of the group configuration by Zilber and Hrushovski, we know that groups abound almost everywhere in 1-based theories.

It has been know for some time [2] that minimal groups are just vector spaces over their (division) ring of definable $*$-endomorphisms (modulo $acl(\emptyset)$) and, more generally, that algebraic dependence of independent generics is connected 1-based groups can be completely described in terms of $*$-endomorphisms.

We show that for certain 1-based groups (i.e. groups in which the only definable set are boolean combinations of cosets of definable subgroups, by [3], in particular totally categorical ones), all structure is determined by the ring of $*$-endomorphisms of some (possibly larger) definable group. While is was know from pp-elimination of quantifiers for modules [1] that all modules are 1-based, Theorem 1 gives a partial converse.

We call an element *reducible* if it is interalgebraic with a finite set of elements of smaller rank. Otherwise it is irreducible. A group is said irreducible if a generic of it is or, equivalently, if all its elements are irreducible.

We show the following:

THEOREM 1. *Let T a unidimensional 1-based theory, and let H be a group whose generics are irreducible with all minimal types being locally finite. Then there is an irreducible group G in H^{eq} containing H as a subgroup such that any connected subgroup of G^n, for $n < \omega$ is the connected component of a subgroup linearly defined over the ring $End^*(G)$. Furthermore, G is generated by H under $End^*(G)$.*

G is essentially a pure QE-module over the ring of definable endomorphisms of G with no extra structure.

[1] W. BAUR, *Elimination of quantifiers for modules*, ***Israel Journal of Mathematics***, vol. 25 (1976), pp. 64–70.

[2] E. HRUSHOVSKI, *Locally modular regular types*, ***Classification theory*** (J. T. Baldwin, editor), Lectures notes in Mathematics, Springer Verlag, 1987, pp. 132–164.

[3] E. HRUSHOVSKI and A. PILLAY, *Weakly normal groups*, ***Logic colloquium '85***, North Holland, Amsterdam-New-York, 1987, pp. 233–244.

► TINKO TINCHEV, *Intersection and graded modalities*.

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As is well known the intersection of two modalities is not expressible. Nevertheless, one can prove that adding to minimal (normal) modal logic the axiom saying that intersection-modality is subset of each of modalities, we obtain a completeness with respect to Kripke models [2]. The detailed analysis in [3] shows that an algorithm exists which for any formula gives its formal proof or tree-like finite model of its negation.

Also, an algorithm exists with the same property, but for graded modalities [4]. Here we mention only that this algorithm is in a sense “orthogonal” to the constraint method of Donini *et al.* [1].

Analyzing these algorithms we find the cause for the difficulties described in [5] for the case of intersection of graded modalities (at least 3): it is needed to add new axioms reflecting the well known combinatorial principle for counting “inclusion and exclusion”. For the obtained formal system about graded modalities with intersections there is an algorithm with the above mentioned property. This gives decidability and completeness with respect to finite tree-like models.

[1] F. DONINI, M. LENZERINI, D. NARDI, and W. NUTT, *The complexity of concept languages*, ***Proceedings of the 2nd conference on knowledge representation***, Morgan Kaufman, San Mateo, California, 1991, pp. 151–162.

[2] G. GARGOV, S. PASSY, and T. TINCHEV, *Modal environment for boolean speculations*,

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[3] T. TINCHEV, *A note on intersection of modalities*, ***Annuaire de l'Universite de Sofia***, Faculté de Mathématiques et Informatique, vol. 87, livre 1 (1993), in print.

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[5] W. VAN DER HOEK and M DE RIJKE, *Counting objects in generalized quantifier theory, modal logic, and knowledge representation*, ***Vrije Universiteit Amsterdam***, Rapportnr. IR-307, 1992.

► SERGEI TUPAILO, *Normalization for the first order classical logic with Hilbert's epsilon-symbol*.
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We prove normalization theorems for three theories: $\mathbf{LK}\varepsilon$, the first order classical logic with Hilbert's epsilon-symbol, $\mathbf{LK}\varepsilon{=}$, $\mathbf{LK}\varepsilon$ + equality, and $\mathbf{LK}\varepsilon{=}\mathbf{E}$, $\mathbf{LK}\varepsilon{=}$ + extensionality.

There were a number of preceding results, proving normalization for different epsilon-calculi. Yasuhara (1982) proved *normal form theorems* (theorems about existence of a normal form) for theories similar to those considered in this paper; Mints (1989) established a normal form theorem for the second order classical logic. Both these proofs use model-theoretic approach. There is also a number of constructive proofs for partial cases. Leisenring (1969) proved normalization theorem for derivations of epsilon-free sequents in a calculus without extensionality. Mints (1973, 1975) gave a proof for the first order intuitionistic systems. Tupailo (1993) established normalization for a weak subsystem of analysis with restricted occurrences of epsilon-symbol. Note that the transfer from intuitionistic systems to classical is not straightforward and requires additional ideas. Still there were no constructive proof of normalization for general case: derivations of arbitrary sequents in a calculus with equality and extensionality.

It turned out to be very useful to formulate rules of inference dealing with epsilon-terms in the form of "generalized cuts", i.e. with a property that each formula in the conclusion occurs in the premises. We take epsilon-rule in the form

$$\frac{\Gamma \longrightarrow \Theta, Ft \qquad F[\varepsilon x F x], \Gamma \longrightarrow \Theta}{\Gamma \longrightarrow \Theta}\ \varepsilon$$

and extensionality rule in the form

$$\frac{\Gamma \longrightarrow \Theta, \forall x(Fx \leftrightarrow Gx) \qquad \varepsilon x F x = \varepsilon x G x, \Gamma \longrightarrow \Theta}{\Gamma \longrightarrow \Theta}\ ext$$

This choice of rules makes standard cut-elimination go through without any changes, and enables to define epsilon-elimination (elimination of non-necessary epsilon-terms) simply as a substitution of t for $\varepsilon x F x$ in an appropriate epsilon-rule, which makes it into a cut. In the case of $\mathbf{LK}\varepsilon{=}\mathbf{E}$, when extensionality is present, the scheme is simple as well: when we are eliminating $\varepsilon x F x$ from a derivation, first $ext-$rules mentioning $\varepsilon x F x$ are to be killed by means of additional cuts (ext is replaced by cuts); after that cuts are eliminated as usual. We hope that this device for eliminating extensionality will be useful in higher order systems.

Skolem's and *Herbrand's theorems* (proved constructively) are obtained as immediate corollaries of normalization.

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It is a well known fact that irreflexivity is not modally definable. In [1] Gabbay shows how to axiomatize irreflexivity by a special rule of inference. In [2] van Benthem noticed that irreflexivity is expressible in a modal language with propositional quantifiers by the formula (Irr) $(\exists p)(\neg p \wedge \Box p)$. We generalize these results for intersection of modalities.

Let $\mathcal{L}$ be a modal language with modalities $\langle\alpha\rangle$, $\langle\beta\rangle$ and $\langle\alpha\cap\beta\rangle$ interpreted in frames of the form $(W, R(\alpha), R(\beta), R(\alpha\cap\beta))$, satisfying the condition $(\cap)$ $R(\alpha\cap\beta) = R(\alpha)\cap R(\beta)$. This condition is not modally definable in $\mathcal{L}$, but, in a language with propositional quantifiers, $(\cap)$ is definable by the following formula

$$\langle\alpha\cap\beta\rangle A \Leftrightarrow (\forall p)(\langle\alpha\rangle(A \wedge p) \vee \langle\beta\rangle(A \wedge \neg p)).$$

Dealing with the axiomatization of the intersection in a non quantified modal language we can simulate the quantifier rule and the Barcan axiom by a special derivation rule, which generalizes Gabbay's irreflexivity rule:

$$\frac{A_0 \Rightarrow [\gamma_1](A_1 \Rightarrow [\gamma_2](A_2 \Rightarrow \dots [\gamma_n](A_n \Rightarrow \langle\alpha\rangle(A \wedge p) \vee \langle\beta\rangle(A \wedge \neg p))\dots))}{A_0 \Rightarrow [\gamma_1](A_1 \Rightarrow [\gamma_2](A_2 \Rightarrow \dots [\gamma_n]A_n \Rightarrow \langle\alpha\cap\beta\rangle A)\dots))}, \quad n = 1, 2, \dots$$

p is a variable, not in $A_1, \dots, A_n, A$ and γ_i is any of α and β.

As an application we can find similar rules for some other constructs as Loop$\alpha = \langle\alpha \cap \mathrm{Id}\rangle$true and complement $\langle-\alpha\rangle A$.

[1] D. M. Gabbay, *An irreflexivity lemma with applications to axiomatizations of conditions on linear frames*, ***Aspects of philosophical logic***, Reidel, Dordrecht, 1981, pp. 67–89.

[2] J. van Benthem, ***Modal logic and classical logic***, Bibliopolis, 1985.

► PAULO A. S. VELOSO, ARMANDO M. HAEBERER AND MARCELO F. FRIAS, *Fork algebras as algebras of logic*.

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Fork algebras are extensions of relational algebras by a *fork* operator, which enables the introduction of projections by definition (making them special *quasi-projective* algebras [2]). Their abstract calculus, motivated by programming needs, provides an algebra for Classical First-Order Logic with Equality (CFOLE), which is finitary [1], in contrast with *Cylindric* and *Polyadic* algebras.

The *Elementary Theory of Fork Relations* (ETFR) is a two-sorted theory, with an injective binary operation $\star$ over sort $\imath$, extending Tarski's *Elementary Theory of Binary Relations* (ETBR) [3] formulated with a ternary predicate symbol -[-,-] over sorts ℓ, ℓ and $\imath$, by a new binary operation ∇ over sort ℓ, with defining axiom

$$(\forall x : \imath)(\forall y : \imath)(r\nabla s[x, y] \leftrightarrow (\exists v : \imath)(\exists w : \imath)(y \approx v \star w \wedge r[x, v] \wedge s[x, w])).$$

Theorem 1 (Expressiveness of ETFR). ETFR *is a conservative extension of* CFOLE, *where for every formula* $\varphi(x_1, \dots, x_n)$ *of* CFOLE *there exists a closed term* r *of sort* ℓ *such that:*

$$\mathsf{ETFR} \vdash (\forall x_1, \dots, x_n : \imath)(\varphi(x_1, \dots, x_n) \leftrightarrow r[\star(x_1, \dots, x_n), \star(x_1, \dots, x_n)]),$$

where $\star(x_1, \dots, x_n) = x_1 \star (x_2 \star \dots (x_{n-1} \star x_n)\dots)$ [3].

The *Abstract Language of Fork Relations* (ALFR) is obtained by forgetting $\imath$, $\star$ and -[-,-]. The *Abstract Calculus of Fork Relations* (ACFR) is obtained by adding to the usual axioms for abstract atomic relation algebras [2] the following axioms

(1) $r\nabla s = (r;\pi) \bullet (s;\rho)$

(2) $(r\nabla s);(p\nabla q)^{\smile} = (r;p^{\smile}) \bullet (s;q^{\smile})$

(3) $\pi\nabla\rho \preceq 1$

(4) $0 \neq t \preceq \infty \otimes \infty \to (\exists v)(\exists w)(0 \neq v \otimes w \preceq t)$

where *projections* and *cross* product are defined by

$$\pi = (1\nabla\infty)^{\smile} \quad \rho = (\infty\nabla 1)^{\smile} \text{ and } r \otimes s = (\pi; r)\nabla(\rho; s).$$

THEOREM 2 (Algebra of Logic). ETFR *is a conservative extension of* ACFR. *For every formula* φ *of* CFOLE *there exists a closed term* $\dagger(\varphi)$ *of* ALFR *such that* CFOLE $\vdash \neg\varphi$ *iff* ACFR $\vdash \dagger(\varphi) = 0$. *This mapping* $\dagger$ *induces a Boolean-algebra isomorphism between the Lindenbaum algebra of formulas of* CFOLE *and the algebra of closed terms of* ALFR *modulo provable equality under* $1 \otimes 1 = 1$.

[1] M. F. FRIAS, G. A. BAUM, A. M. HAEBERER, and P. A. S. VELOSO, *A representation theorem for fork-algebras*, *PUC-Rio Res. Rept.*, MCC 29, 1993, To appear in: ***Bulletin of the Section of Logic, Polish Academy of Sciences***.

[2] A. TARSKI and S. GIVANT, *A formalization of set theory without variables*, ***American Mathematical Society Colloquium Publication***, vol. 41 (1987).

[3] P. A. S. VELOSO and A. M. HAEBERER, *A finitary relational algebra for classical first-order logic*, ***Bulletin of the Section of Logic, Polish Academy of Sciences***, vol. 20 (1991), no. 2, pp. 52–62.

► ANDREAS WEIERMANN, *How to characterize provably total functions by local predicativity*.

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The aim of this paper is to give a straightforward characterization of the provably total functions of Peano-Arithmetic, of Peano-Arithmetic plus the scheme of transfinite induction over initial segments of certain primitive recursive well-orderings and of Kripke Platek set theory with the axiom of infinity. Of course, our results are standard results from classical proof theory.

However, not only experts in proof theory and subrecursive hierarchies but also a wider readership interested in general proof-theoretic techniques will find this paper interesting, since we avoid completely the use of heavy metamathematical machinery like e.g. Mints' continuous cut elimination. Up to now the most direct approach of characterizing provably total functions of Peano-Arithmetic in terms of the fast growing hierarchy below level ε_0 was proposed in [Buchholz and Wainer (1987)]. Compared with this reference this paper's approach has the following advantages. Based on a new approach to subrecursion which is developed in [Buchholz, Cichon and Weiermann (1994)] we develop a very smooth theory of deduction relations $<_n$. These have much nicer closure-properties (for example compatibility with the natural sum) than the relations $\prec_n$ used in [Buchholz and Wainer (1987)]. The verification of these properties can be done by simple one-line-proofs. Moreover this theory can be generalized straightforwardly to stronger ordinal notation systems without much complication. As an immediate corollary to the usual cut elimination procedure for infinitary derivations we obtain the desired result by local predicativity style methods [cf. Pohlers (1981)] in a very perspicuous way. Also, our approach may be applied with minor modifications uniformly to very strong theories. To indicate this we apply our methods to $KP\omega$. In the meantime the author's idea has been successfully applied to the theory KPM, the strongest theory for which an ordinal analysis has been published to-date (cf. [Rathjen (1991)]).

► A. D. YASHIN, *The Smetanich logic has the unique Novikov complete extension*.

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Let B the intuitionistic propositional logic, $L(\varphi)$ be the extension of the intuitionistic propositional language by a new propositional constant symbol φ and $H(\varphi)$ is a logic in the language $L(\varphi)$ such thay $H \subseteq H(\varphi)$.

Definition 1 (P. Novikov). $H(\varphi)$ *determines* a new intuitionistic logical constant φ iff

(1) $H(\varphi)$ is a conservative extension of H;
(2) for every formula B without occurrences of φ, the logic $H(\varphi) + \{\varphi \equiv B\}$ is not conservative over H.

DEFINITION 2 (P. Novikov). $H(\varphi)$ is said to be *complete* iff

(1) $H(\varphi)$ is a conservative extension of H;
(2) for every formula $B \notin H(\varphi)$, the logic $H(\varphi) + \{B\}$ is not conservative over H.

EXAMPLE (Smetanich [1]). The logic $Sm = H + \{\neg\neg\varphi, \varphi \supset (p \vee \neg\varphi)\}$ determines a new intuitionistic logical constant.

It is clear (by Zorn Lemma) that every conservative over H logic $H(\varphi)$ has a Novikov completion.

The *Novikov's problem* was formulated ([1]) as follows: to give an explicit example of a complete logic which determines a new intuitionistic logical constant.

THEOREM. *Sm has the unique Novikov complete extension. This completion is finitely axiomatizable, it is characterized by a simple class of finite tree frames, and it is decidable.*

[1] YA. S. SMETANICH, *On completeness of the propositional calculus with additional unary operation*, ***Proceedings of Moscow Mathematical Society***, vol. 9 (1960), pp. 357–371.

► MICHAEL ZAKHARYASCHEV AND ALEXANDER ALEKSEEV, *The decidability of finitely axiomatizable normal modal logics containing K4.3*.

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By the well known results of Bull and Fine, all modal logics above **S4.3** have the finite model property, finite axiomatizations and so are decidable. However, normal extensions of **K4.3**, also characterized by Kripke frames of width 1 without infinite ascending chains (i.e., by *Noetherian chains of clusters*), but possibly with irreflexive points, do not necessarily have the finite model property and finite axiomatizations and may be undecidable (see Kracht [1]). Yet, by using the canonical formulas of Zakharyaschev [2], we show that finitely axiomatizable normal logics above **K4.3** are decidable. All the frames below are assumed to be Noetherian chains of clusters; NExt**K4.3** denotes the set of all normal extensions of **K4.3**.

THEOREM 1. *For every frame $\mathbb{G}$ and every canonical formula $\alpha(\mathbb{F}, \mathbb{D}, \bot)$, $\mathbb{G} \not\models \alpha(\mathbb{F}, \mathbb{D}, \bot)$ iff there is an injective cofinal subreduction of $\mathbb{G}$ to $\mathbb{F}$ satisfying the closed domain condition (CD) for $\mathbb{D}$.*

In other words, every refutation frame for $\alpha(\mathbb{F}, \mathbb{D}, \bot)$ can be obtained from $\mathbb{F}$ by inserting some Noetherian chains below clusters $C(x) \in \mathbb{F}$ generated by points x such that $\{x\} \notin \mathbb{D}$ and possibly by enlarging some non-degenerate clusters in $\mathbb{F}$.

Let $\alpha(\mathbb{F}, \mathbb{D}, \bot)$ be a canonical formula, $\mathbb{F} = \langle W, R \rangle$ and $C(x_0)R \ldots RC(x_n)$ all distinct clusters in $\mathbb{F}$. By a *type for* $\alpha(\mathbb{F}, \mathbb{D}, \bot)$ we mean any n–tuple $t = \langle \xi_1, \ldots, \xi_n \rangle$ such that, for $i \in \{1, \ldots, n\}$, either $\xi_i = m$ or $\xi_i = m+$, for some $m < \omega$, or $\xi_i = \omega$, with $\xi_i = 0$ if $\{x_i\} \in \mathbb{D}$. Given a type $t = \langle \xi_1, \ldots, \xi_n \rangle$ for $\alpha(\mathbb{F}, \mathbb{D}, \bot)$, we define a *t-extension* of $\mathbb{F}$ as the frame $\mathbb{G}$ that is obtained from $\mathbb{F}$ by inserting between each pair $C(x_{i-1})$, $C(x_i)$ either a descending chain of m irreflexive points, if $\xi_i = m < \omega$, or a descending chain of $m + 1$ points of which only the last (lowest) one is reflexive, if $\xi_i = m+$, or an infinite descending chain of irreflexive points, if $\xi_i = \omega$.

THEOREM 2. *If $L \in$ NExt**K4.3** and $\alpha(\mathbb{F}, \mathbb{D}, \bot) \notin L$, then there is a type t for $\alpha(\mathbb{F}, \mathbb{D}, \bot)$ such that the t-extension of $\mathbb{F}$ validates L and refutes $\alpha(\mathbb{F}, \mathbb{D}, \bot)$.*

It follows that every logic in NExt**K4.3** is characterized by a recursively enumerable set of (possibly infinite) frames.

Let $\mathbb{H}$ be a finite frame, k the number of irreflexive points in $\mathbb{H}$, $t = \langle \xi_1, \dots, \xi_n \rangle$ a type for $\alpha(\mathbb{F}, \mathbb{D}, \bot)$ and $\mathbb{G}$ the t-extension of $\mathbb{F}$. Construct a subframe $\mathbb{G}_k$ of $\mathbb{G}$ by "cutting off" the infinite descending chains inserted in $\mathbb{F}$ just below their $k+1$th points and let X be the set of all these $k+1$th points.

THEOREM 3. *$\mathbb{G} \not\models \alpha(\mathbb{H}, \mathbb{E}, \bot)$ iff there is an injective cofinal subreduction f of $\mathbb{G}_k$ to $\mathbb{H}$ satisfying (CD) for $\mathbb{E}$ and such that $X \cap \operatorname{dom} f = \emptyset$.*

Thus, given a type t for $\alpha(\mathbb{F}, \mathbb{D}, \bot)$ and a canonical formula $\alpha(\mathbb{H}, \mathbb{E}, \bot)$, only finitely many steps is required to verify whether the t-extension of $\mathbb{F}$ refutes $\alpha(\mathbb{H}, \mathbb{E}, \bot)$, and so we can prove the decidability of finitely axiomatizable logics in NExt**K4.3** by using Harrop's old good argument.

THEOREM 4. *All finitely axiomatizable normal extensions of **K4.3** are decidable.*

[1] M. KRACHT, *Prefinitely axiomatizable modal and intermediate logics*, ***Mathematical Logic Quarterly***, vol. 39 (1993), pp. 301–322.

[2] M. ZAKHARYASCHEV, *Canonical formulas for **K4**, Part I: Basic results*, ***The Journal of Symbolic Logic***, vol. 57 (1992), pp. 1377–1402.