HW9 - Problems

Andrea Brose, Math 167/1, F05

Due 2. December 2005

AP(xix) Prove proposition 16 from lecture on 18.11.2005.
Proposition 16 If $X \subseteq \mathbb{R}^n$ is bounded (see definition 43) then X is bounded above.

AP(xx) Prove Lemma 8 from lecture on 23.11.2005.
Lemma 8 Convex combinations are invariant under affine transformations, i.e. given an affine transformation $\tau : \mathbb{R}^n \to \mathbb{R}^n$, then

$$\tau\left(\sum_{i=1}^{k} p_i \boldsymbol{x}_i\right) = \sum_{i=1}^{k} p_i \tau(\boldsymbol{x}_i)\,,$$

where $p_i \in \mathbb{R}_0^+$, $i = 1, \ldots, k$ such that $\sum_{i=1}^{k} p_i = 1$.

AP(xxi) Prove lemma 10 from lecture on 23.11.2005.
Lemma 10 Supporting lines are invariant under affine transformations, i.e. given a convex set $X \subseteq \mathbb{R}^2$, $\boldsymbol{s} \in \partial X$, an affine transformation $\tau : \mathbb{R}^2 \to \mathbb{R}^2$, and a supporting line l to X at $\boldsymbol{s}$, then

(a) τX is convex set,

(b) $\tau \boldsymbol{s} \in \partial(\tau X)$, and

(c) τl is supporting line to τX at $\tau \boldsymbol{s}$.

AP(xxii) **Lemma 11** Under certain conditions affine transformations are invertible.

(a) Name condition(s) (try to make them as minimal as possible) under which affine transformations are invertible.

(b) Show that under your stated condition(s), affine transformations are indeed invertible.

AP(xxiii) Prove the following 'technicalities' from the proof of theorem 14 from lecture on 28.11.2005.
Given a Nash Bargaining Problem $(X, \boldsymbol{d})$, where $\boldsymbol{d} = (d_1, d_2)$ and utilities u and v for players I and II respectively, define

$$u' = \frac{u - d_1}{x - d_1} \quad \text{and} \quad v' = \frac{v - d_2}{y - d_2}$$

then

(a) $\tau(u, v) = (u', v')$ is affine transformation.

(b) $\tau\boldsymbol{d} = (0, 0)$

(c) $\tau\boldsymbol{r} = (1, 0)$

(d) $\tau\boldsymbol{t} = (0, 1)$